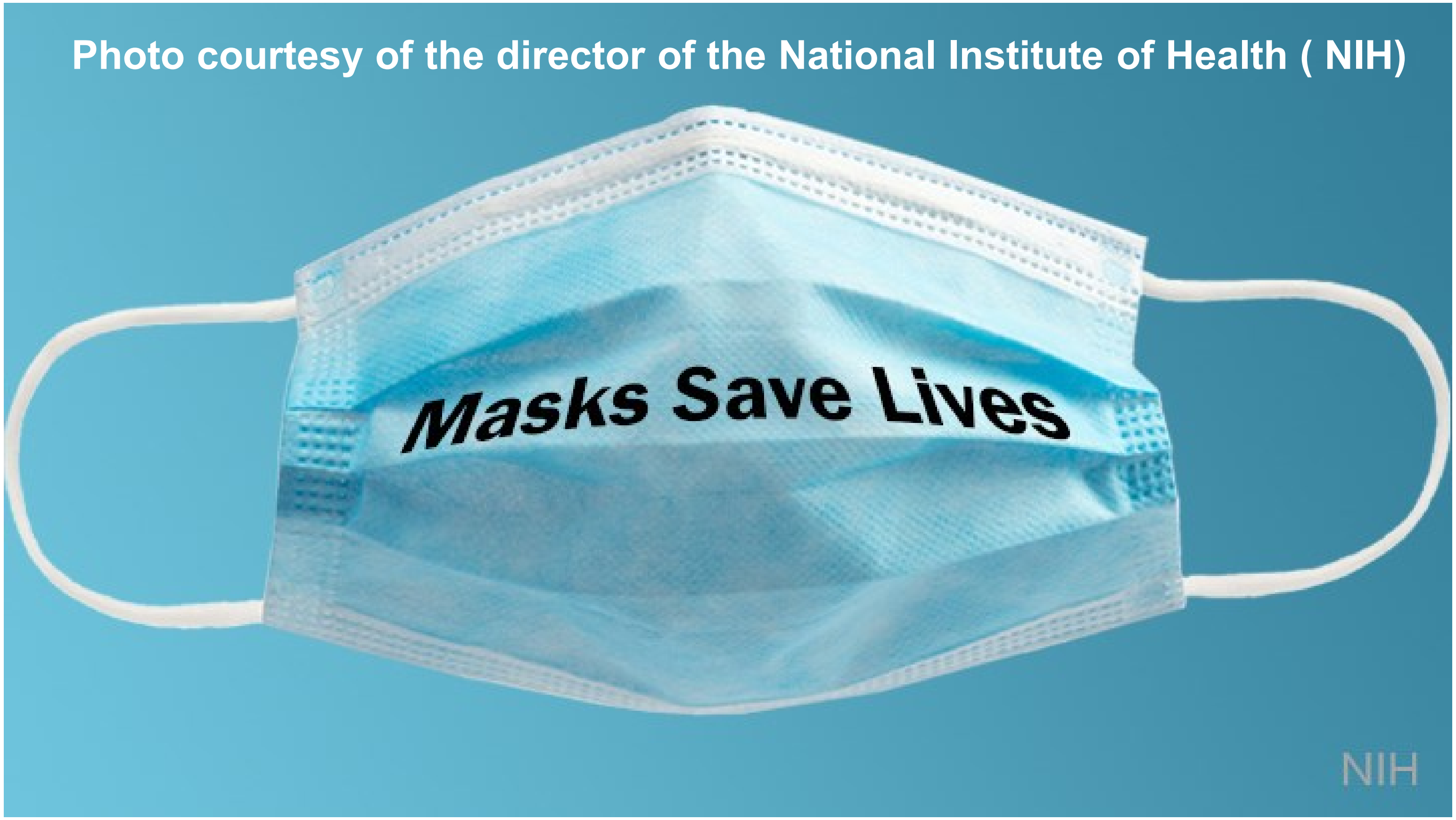
# EE 330
# Lecture 4

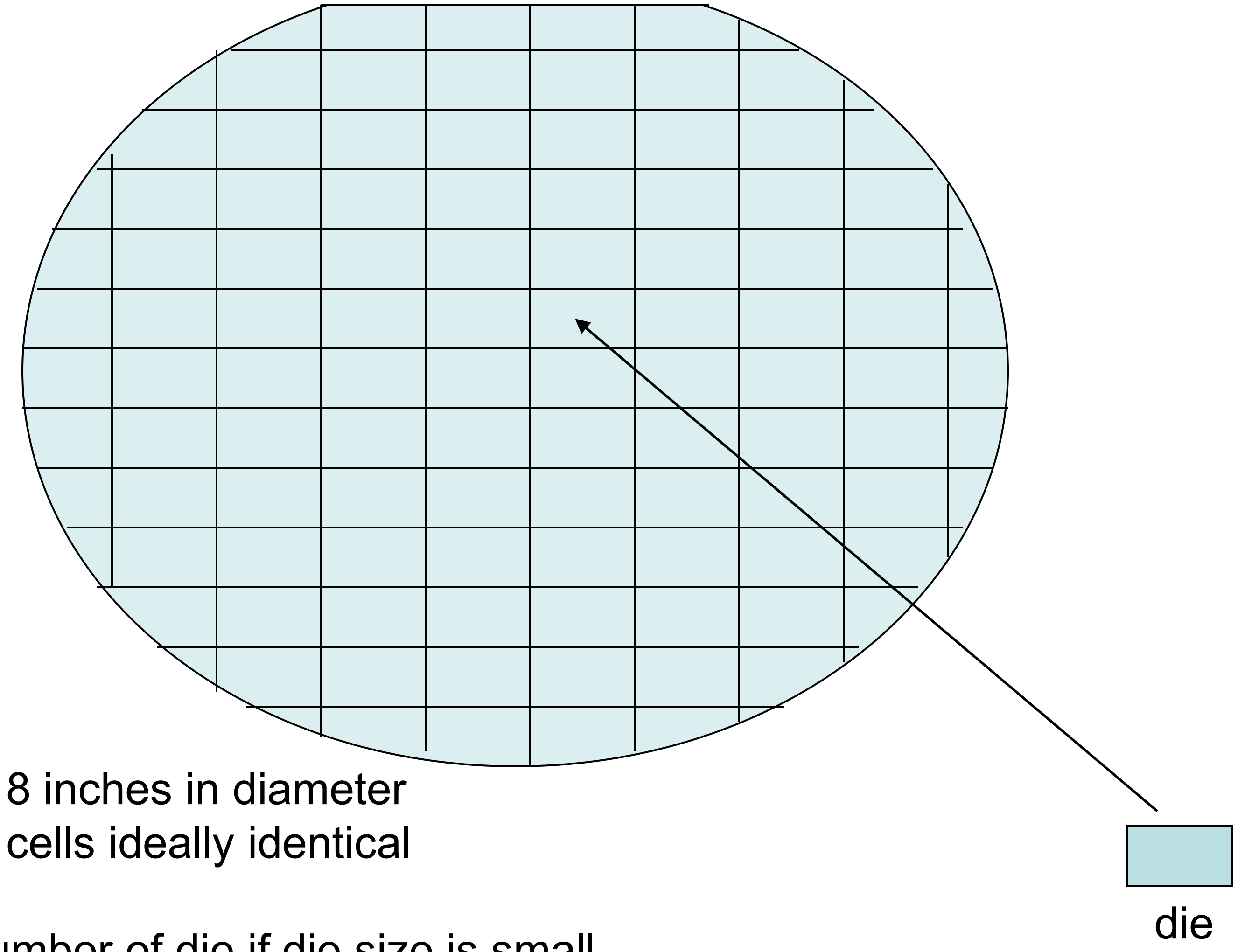
- Yield
- Statistics Review




As a courtesy to fellow classmates, TAs, and the instructor

Wearing of masks during lectures and in the laboratories for this course would be appreciated irrespective of vaccination status

# Wafer



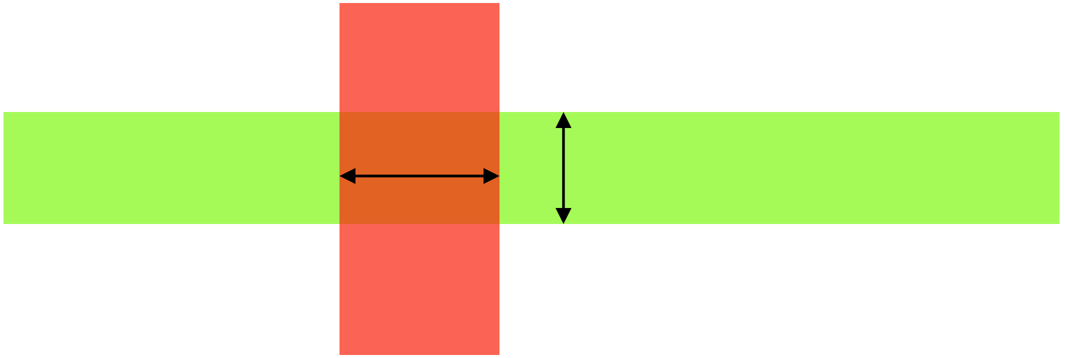

- 6 inches to 18 inches in diameter
- All complete cells ideally identical
- flat edge
- very large number of die if die size is small

Review from Last Time

# Feature Size

Feature size is the minimum lateral feature size that can be **reliably** manufactured


Often given as either feature size or pitch

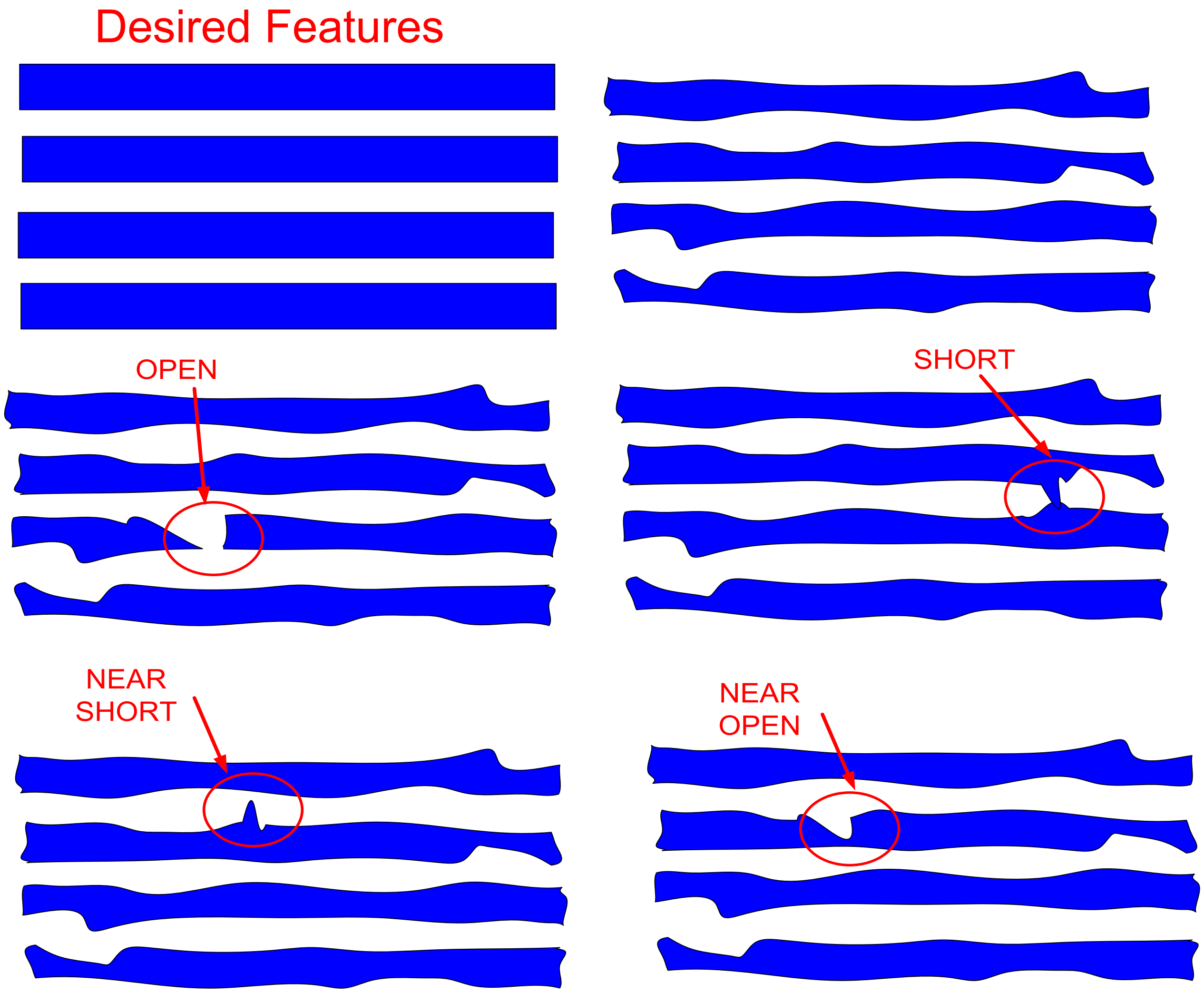


Minimum feature size often identical for different features

Extremely challenging to decrease minimum feature size in a new process

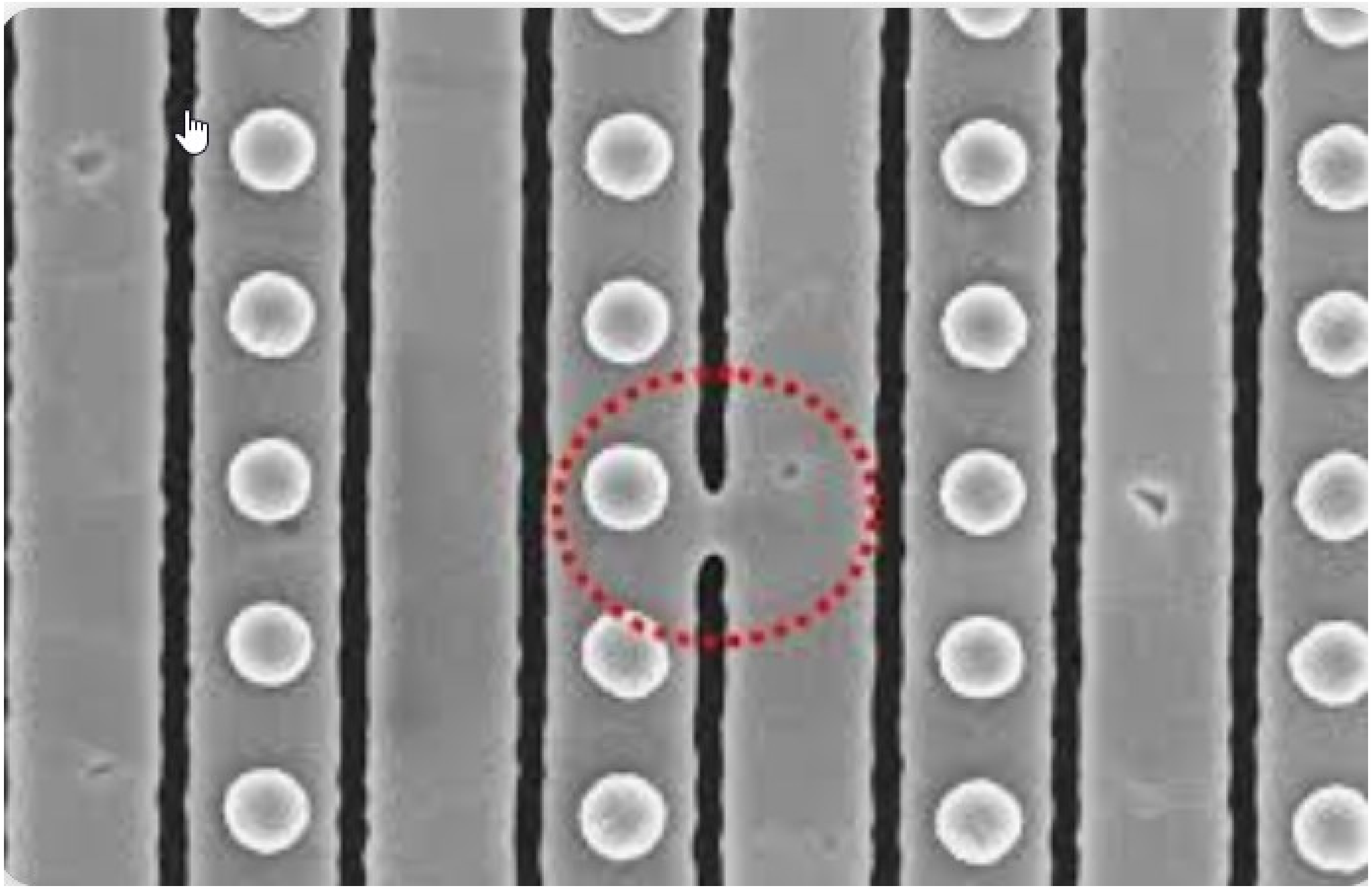
Review from Last Time

# Reliability Problems




Actual features show some variability (dramatically exaggerated here !!!!)

# SEM Images of Irregularity and/or Defects



Review from Last Time

# What is meant by "reliably"

Yield is acceptable if circuit performs as designed even when a very large number of these features are made

If P is the probability that a feature is good

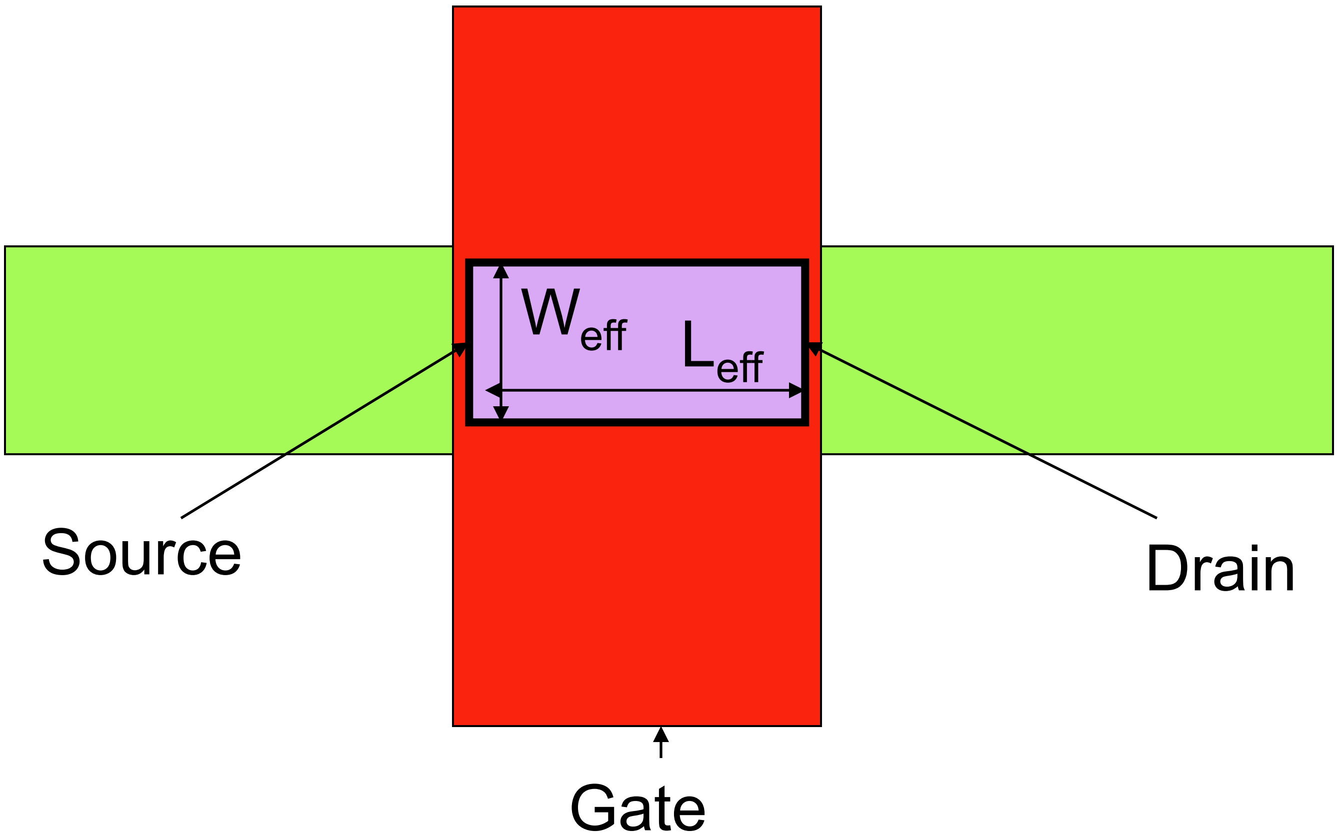
n is the number of uncorrelated features on an IC

Y is the yield

$$Y = P^n$$

$$P = e^{\frac{\log_e Y}{n}}$$

# MOS Transistor




Effective Width and Length Generally Smaller than Drawn Width and Length

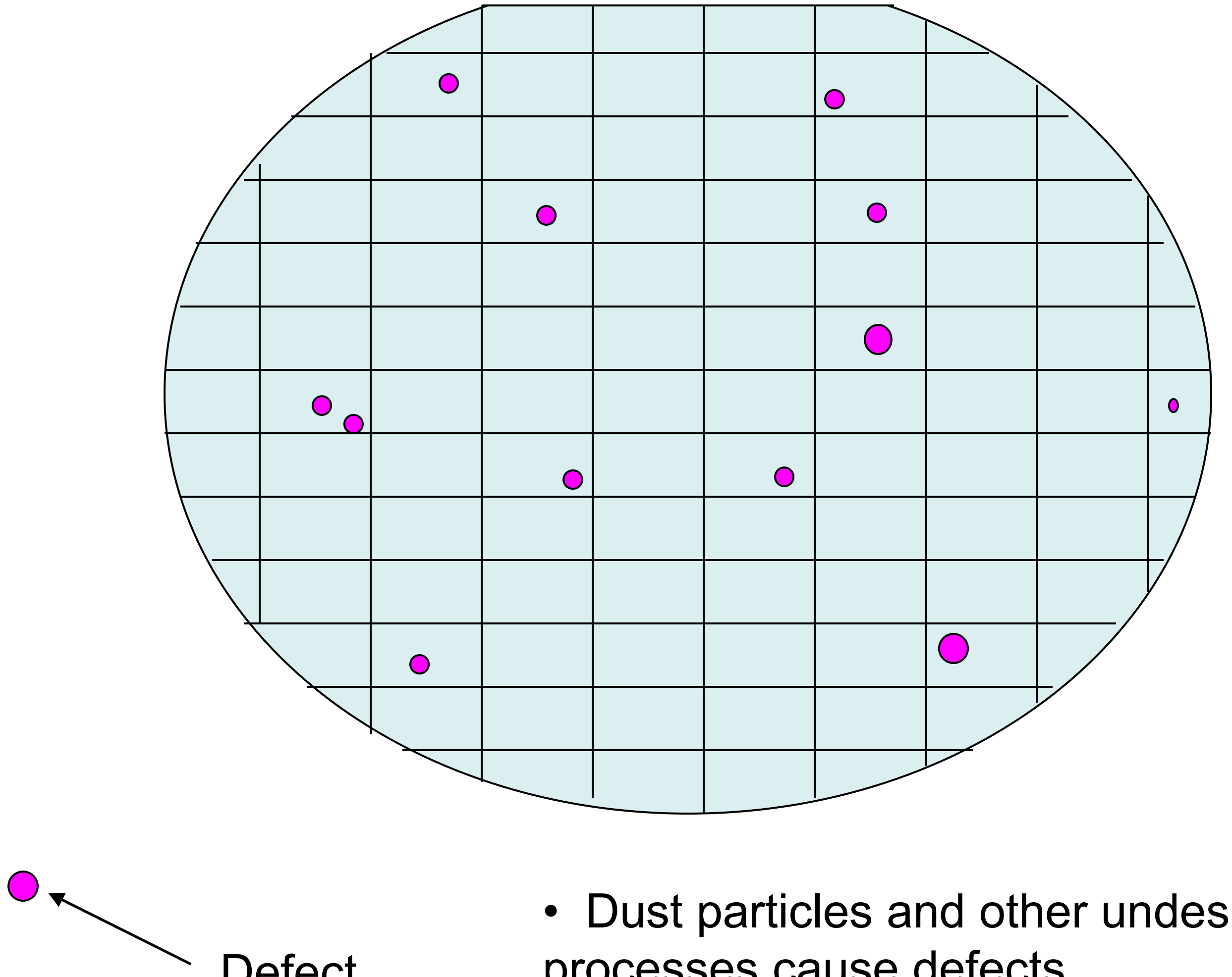
Review from Last Time

# Size of Atoms and Molecules in Semiconductor Processes

| | | |
|---|---|---|
| Silicon: | Average Atom Spacing | $2.7\,\overset{\text{o}}{\text{A}}$ |
| | Lattice Constant | $5.4\,\overset{\text{o}}{\text{A}}$ |
| $S_iO_2$ | Average Atom Spacing | $3.5\,\overset{\text{o}}{\text{A}}$ |
| | Breakdown Voltage | $5 \text{ to } 10\text{MV/cm} = 5 \text{ to } 10\text{mV/}\overset{0}{\text{A}}$ |
| Air | | $20\text{KV/cm}$ |

Physical size of atoms and molecules place fundamental limit on conventional scaling approaches

# Defects in a Wafer




- Dust particles and other undesirable processes cause defects
- Defects in manufacturing cause yield loss

# Yield Issues and Models

- Defects in processing cause yield loss
- The probability of a defect causing a circuit failure increases with die area
- The circuit failures associated with these defects are termed **Hard Faults**
- This is the major factor limiting the size of die in integrated circuits
- Wafer scale integration has been a “gleam in the eye” of designers for 3 decades but the defect problem continues to limit the viability of such approaches
- Several different models have been proposed to model the hard faults

# Yield Issues and Models

- Parametric variations in a process can also cause circuit failure or cause circuits to not meet desired performance specifications (this is of particular concern in analog and mixed-signal circuits)
- The circuits failures associated with these parametric variations are termed **Soft Faults**
- Increases in area, judicious layout and routing, and clever circuit design techniques can reduce the effects of soft faults

# Hard Fault Model

$$Y_H = e^{-Ad}$$

$Y_H$ is the probability that the die does not have a hard fault
A is the die area
d is the defect density (typically $1\text{cm}^{-2} < d < 2\text{cm}^{-2}$)

Industry often closely guards the value of d for their process

Other models, which may be better, have the same general functional form

# Soft Fault Model

Soft fault models often dependent upon design and application

Often the standard deviation of a parameter is dependent upon the reciprocal of the square root of the parameter sensitive area

$$\sigma = \frac{\rho}{\sqrt{A_k}}$$

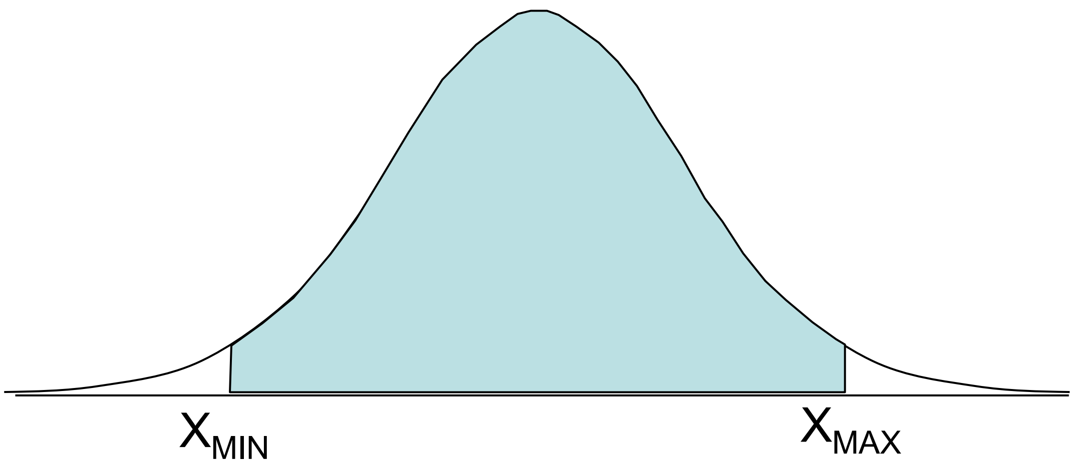
$\rho$ is a constant dependent upon the architecture and the process

$A_k$ is the area of the parameter sensitive area

# Soft Fault Model

$$P_{SOFT} = \int_{X_{MIN}}^{X_{MAX}} f(x)\,dx$$

$P_{SOFT}$ is the soft fault yield
$f(x)$ is the probability density function of the parameter of interest
$X_{MIN}$ and $X_{MAX}$ define the acceptable range of the parameter of interest




Some circuits may have several parameters that must meet performance requirements

# Soft Fault Model

If there are k parameters that must meet parametric performance requirements and if the random variables characterizing these parameters are uncorrelated, then the soft yield is given by

$$Y_S = \prod_{j=1}^{k} P_{SOFT_j}$$

# Overall Yield

If both hard and soft faults affect the yield of a circuit, the overall yield is given by the expression

$$Y = Y_H Y_S$$

# Cost Per Good Die

The manufacturing costs per good die is given by

$$C_{Good} = \frac{C_{FabDie}}{Y}$$

where $C_{FabDie}$ is the manufacturing costs of a fab die and Y is the yield

There are other costs that must ultimately be included such as testing costs, engineering costs, packaging costs,  etc.

Example: Assume a die has no soft fault vulnerability, a die area of $1cm^2$ and a process has a defect density of $1.5cm^{-2}$

a) Determine the hard yield
b) Determine the manufacturing cost per good die if 8” wafers are used and if the cost of the wafers is $1200

# Solution

a) $$Y_H = e^{-Ad}$$

$$Y = e^{-1\text{cm}^2 \bullet 1.5\text{cm}^{-2}} = 0.22$$

b) $$C_{Good} = \frac{C_{FabDie}}{Y}$$

$$C_{FabDie} = \frac{C_{Wafer}}{A_{Wafer}} A_{Die}$$

$$C_{FabDie} = \frac{\$1200}{\pi(4\text{in})^2} 1\text{cm}^2 = \$3.82$$

$$C_{Good} = \frac{\$3.82}{0.22} = \$17.37$$

Do you like statistics ?

# Statistics are Real!

Statistics govern what really happens throughout much of the engineering field!

Statistics are your Friend !!!!

You might as well know what will happen since statistics characterize what WILL happen in the presence of variability in many processes !

# Statistics Review

Assume x is a random variable of interest

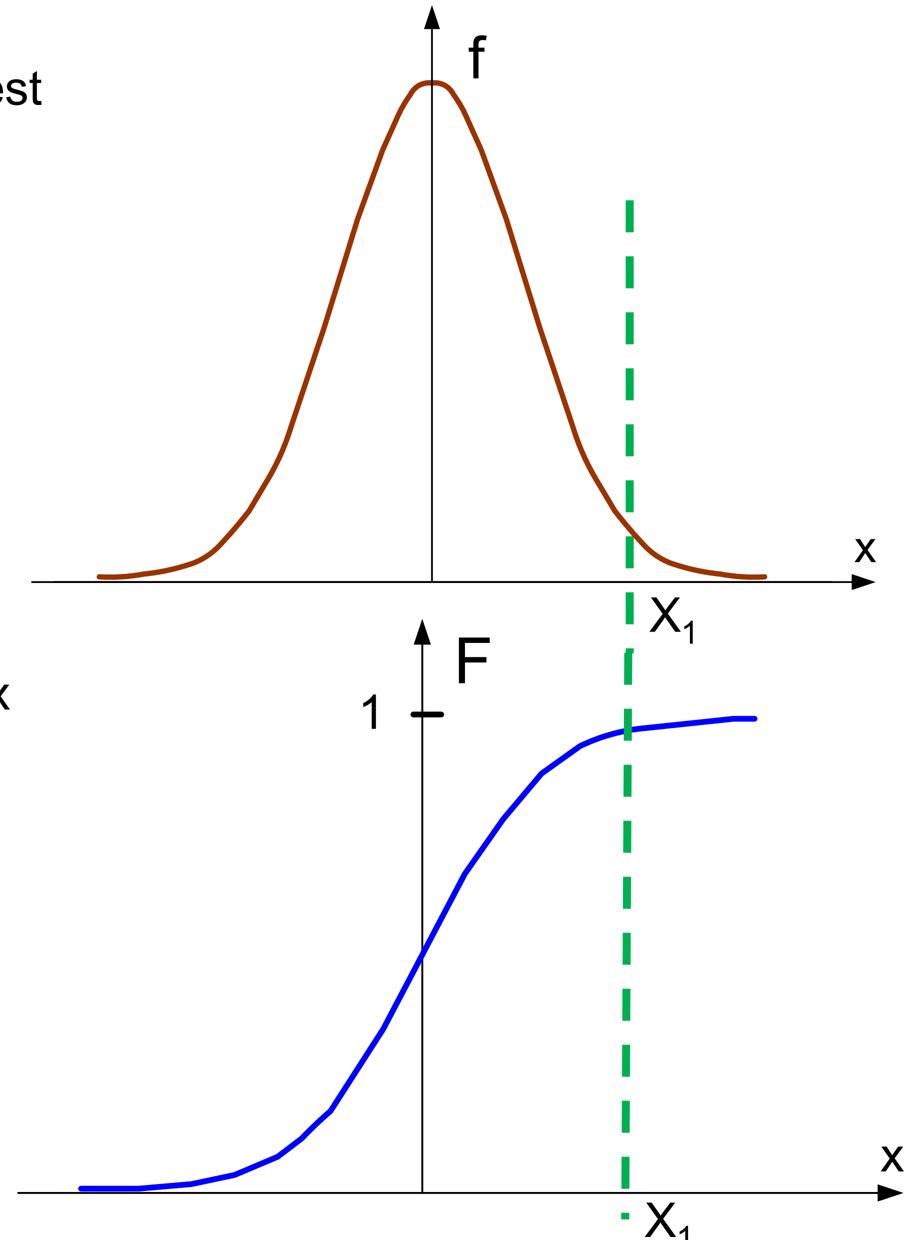
f(x) = Probability Density Function for x

$$\int_{x=-\infty}^{\infty} f(x)\,dx = 1$$

F(x) = Cumulative Density Function for x

$$F(X_1) = \int_{x=-\infty}^{X_1} f(x)\,dx$$

$$0 \leq F(x) \leq 1 \qquad \frac{\partial F(x)}{\partial x} \geq 0$$



# Statistics Review

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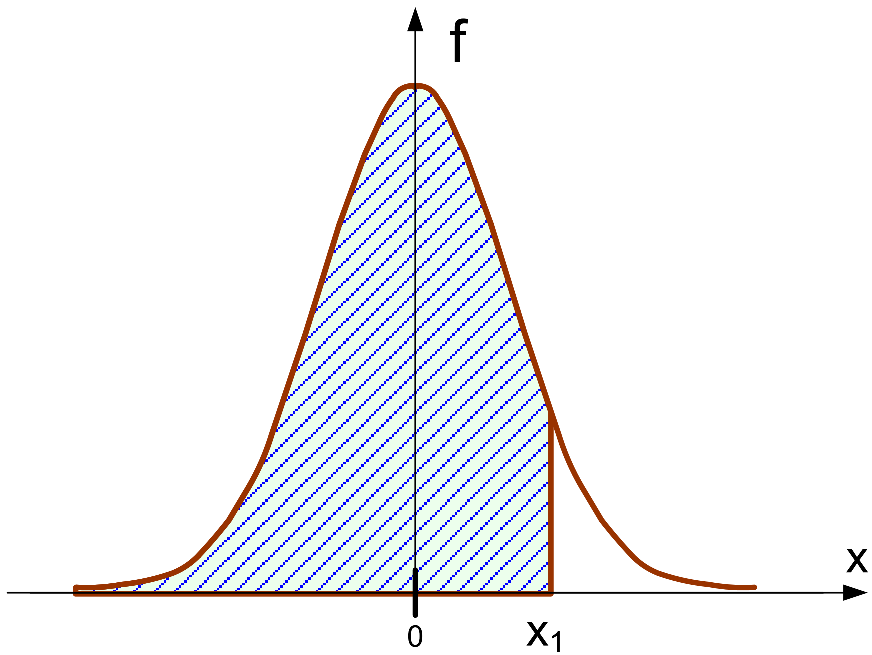
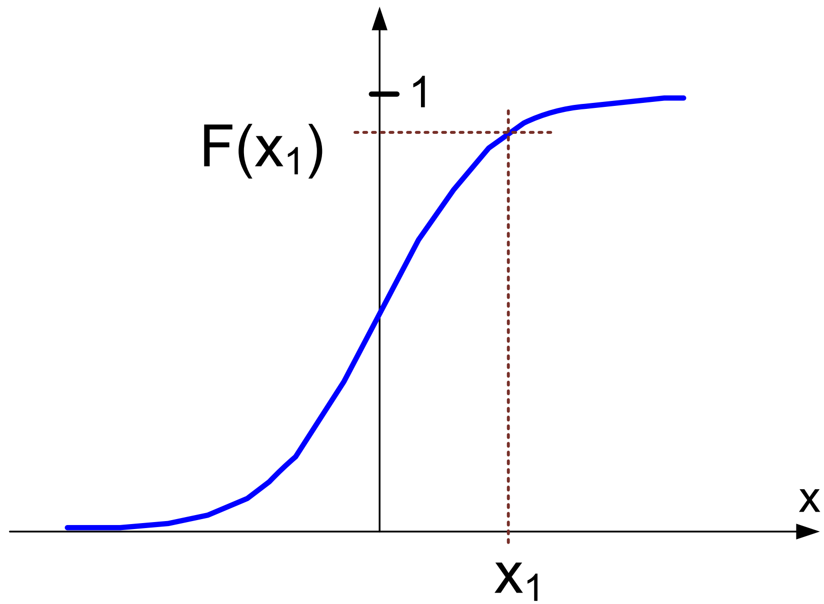



$$P\{x \le x_1\} = \int_{x=-\infty}^{x_1} f(x)\,dx$$

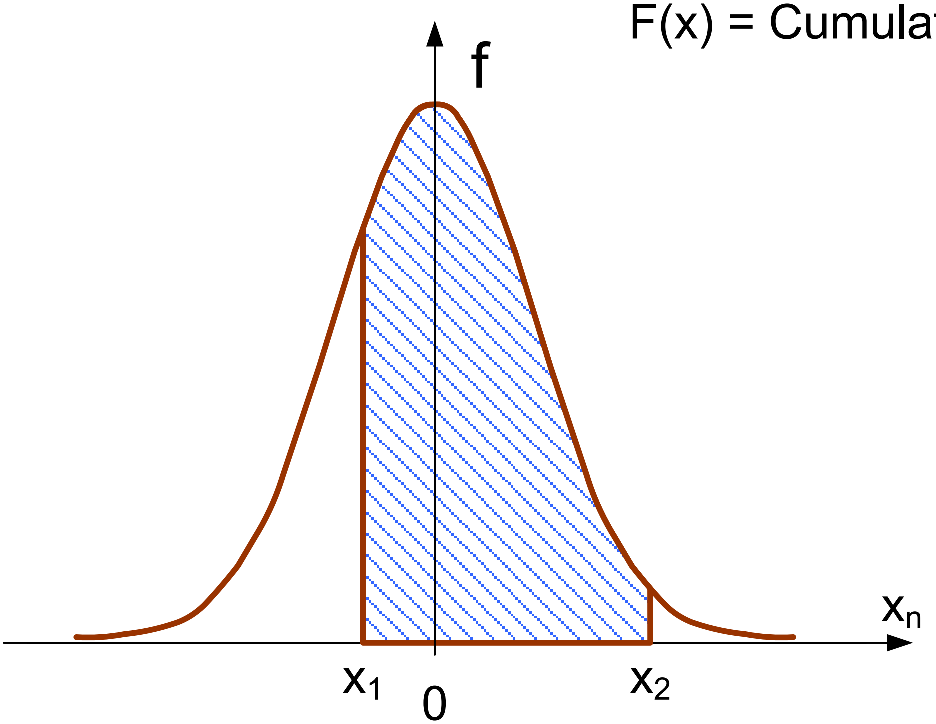
$$P\{x \le X_1\} = F(X_1)$$

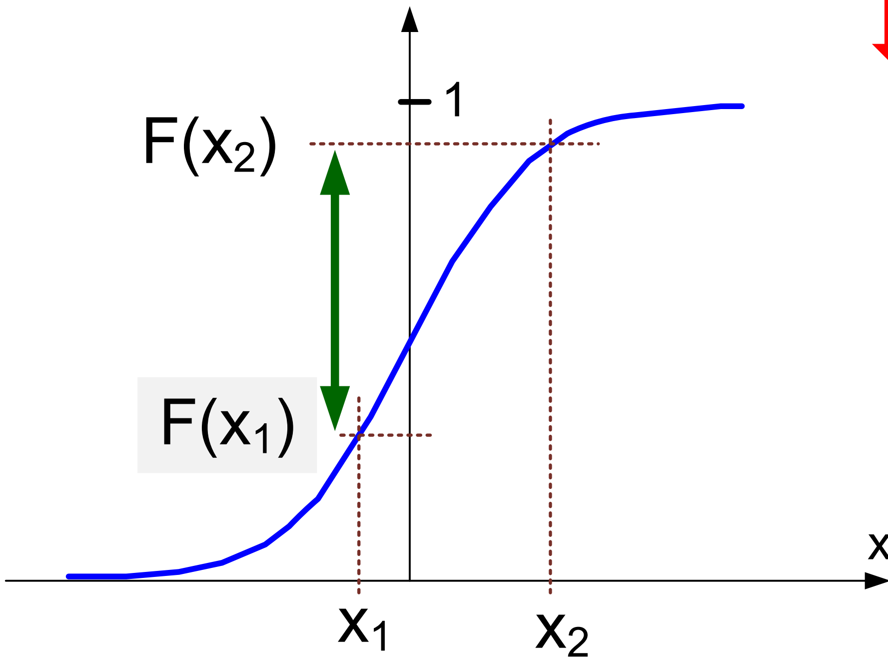
$$F(x_1) = \int_{x=-\infty}^{x_1} f(x)\,dx$$

# Statistics Review

f(x) = Probability Density Function for x

F(x) = Cumulative Density Function for x

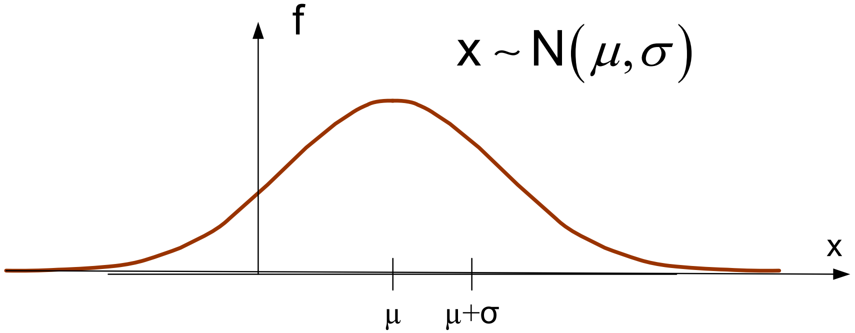


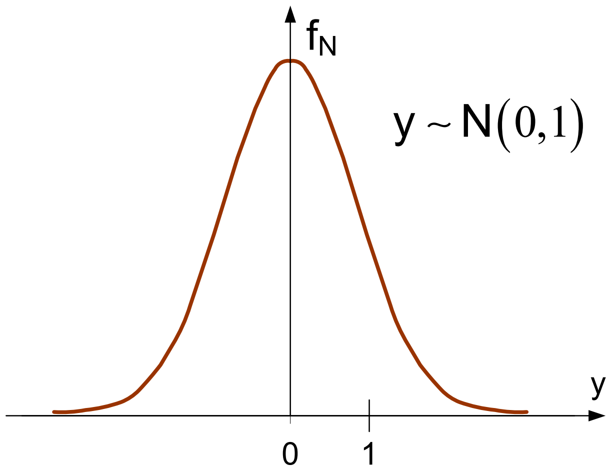




$$P\{X_1 \le x \le X_2\} = \int_{X_1}^{X_2} f(x)\,dx$$

$$P\{X_1 \le x \le X_2\} = F(X_2) - F(X_1)$$

# Statistics Review




$$\int_{x=-\infty}^{\infty} f(x)\,dx = 1$$

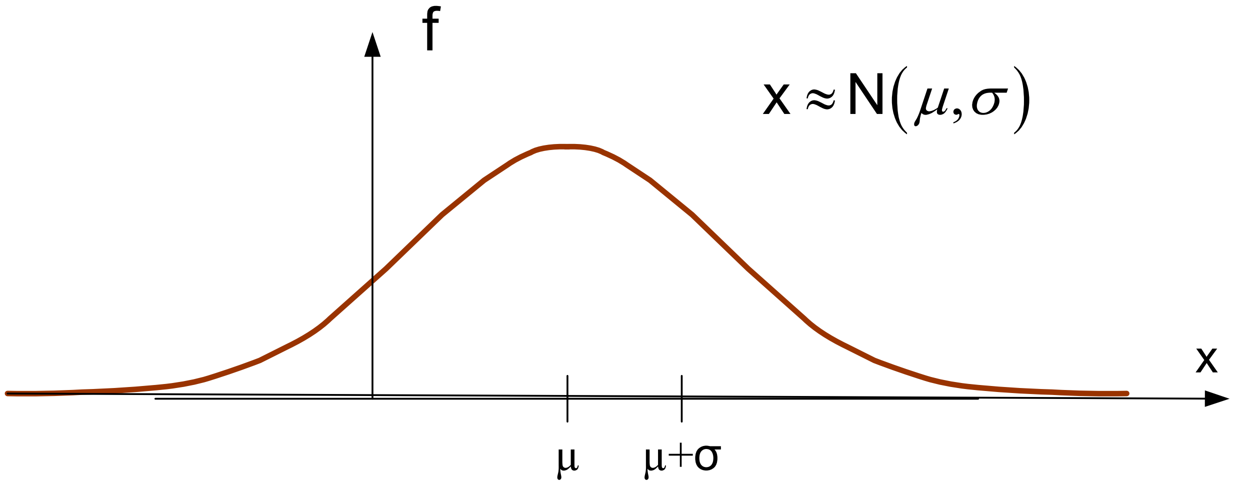
$$y = \frac{x-\mu}{\sigma}$$

$$\int_{y=-\infty}^{\infty} f_N(y)\,dy = 1$$

Theorem 1: If the random variable x in normally distributed with mean μ and standard deviation σ, then $y = \frac{x-\mu}{\sigma}$ is also a random variable that is normally distributed with mean 0 and standard deviation of 1.

(Normal Distribution and Gaussian Distribution are the same)

# Statistics Review

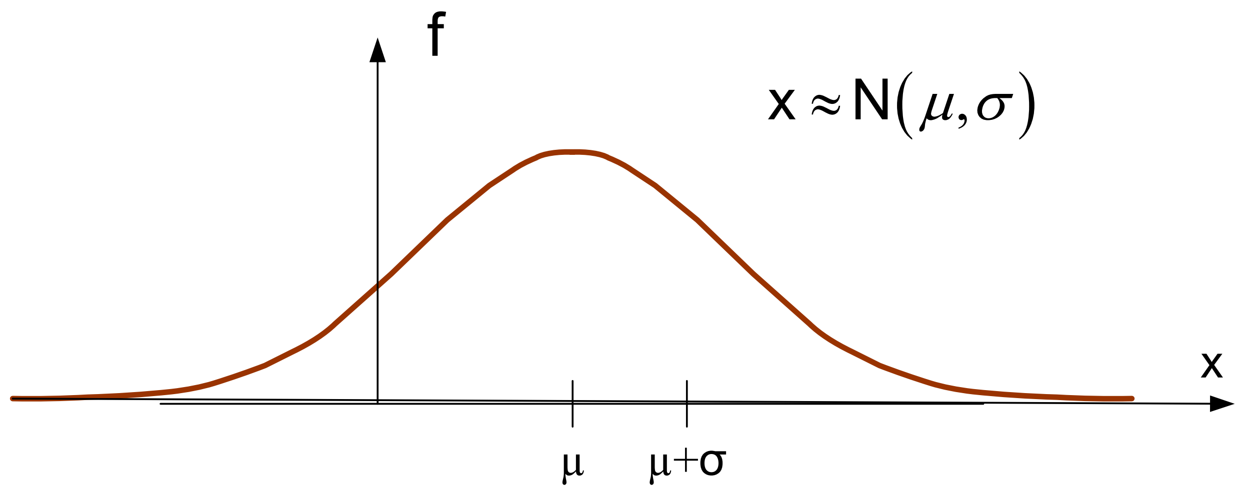



The random part of many parameters of microelectronic circuits is often assumed to be Normally distributed and experimental observations confirm that this assumption provides close agreement between theoretical and experimental results

The mapping $y = \dfrac{x-\mu}{\sigma}$ is often used to simplify the statistical characterization of the random parameters in microelectronic circuits

x generally is dimensioned, y is dimensionless

# Statistics Review




Example:

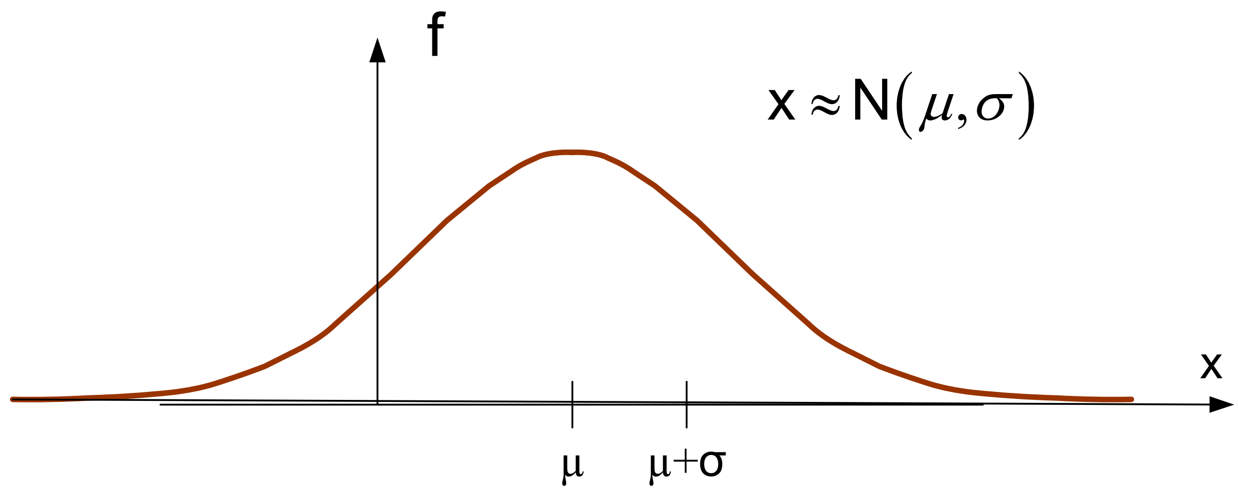
x might be the frequency of oscillation of a ring oscillator used for a clock in a crystal-less digital circuit, x Gaussian (Normal)

Dimensions of x : Hz
Maybe $\mu$=550 MHz $\sigma$=50 MHz

$$y = \frac{x-\mu}{\sigma}$$ is dimensionless with $\mu_y=0$ $\sigma_y=1$

y: N(0,1)

# Statistics Review




Example:

x might be the offset voltage of an op amp, x Gaussian (Normal)

Dimensions of x : Volts
Typically $\mu$=0V $\sigma$=10 mV

$$y = \frac{x - \mu}{\sigma}$$ is dimensionless with $\mu_y=0$ $\sigma_y=1$

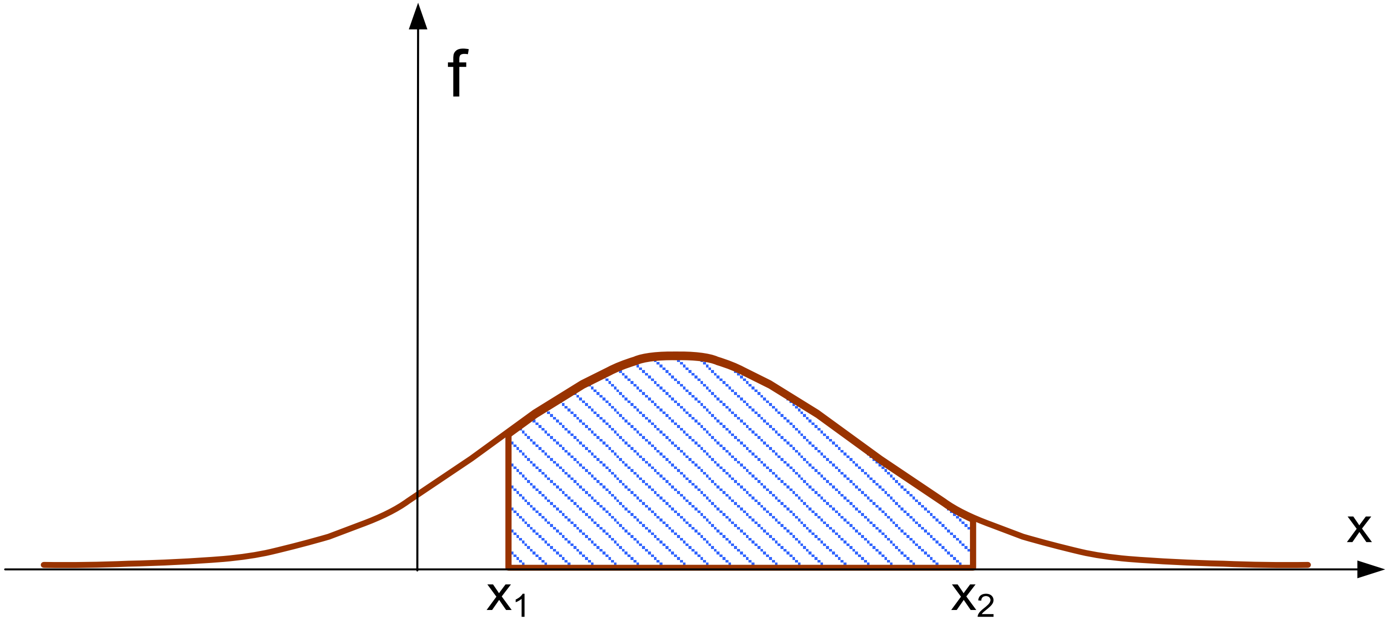
y: N(0,1)

*Background Information*

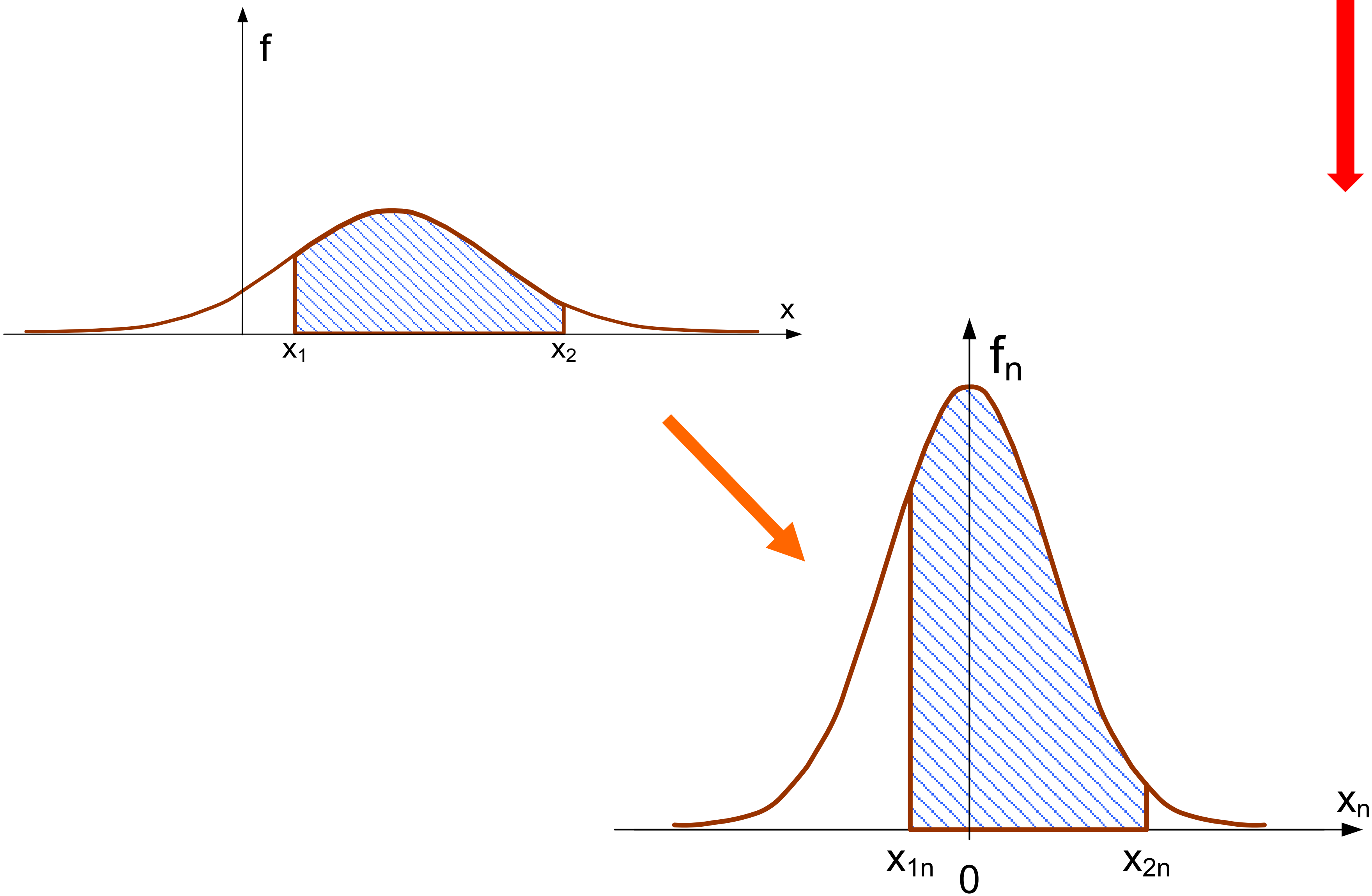
*Theorem 2: If x is a Normal (Gaussian) random variable with mean μ and standard deviation σ, then the probability that x is between $x_1$ and $x_2$ is given by*

$$p = \int_{x_1}^{x_2} f(x)\,dx = \int_{x_{1n}}^{x_{2n}} f_n(x)\,dx \quad \text{where} \quad x_{1n} = \frac{x_1 - \mu}{\sigma} \quad \text{and} \quad x_{2n} = \frac{x_2 - \mu}{\sigma}$$

and where $f_n(x)$ is N(0,1)



*Background Information*

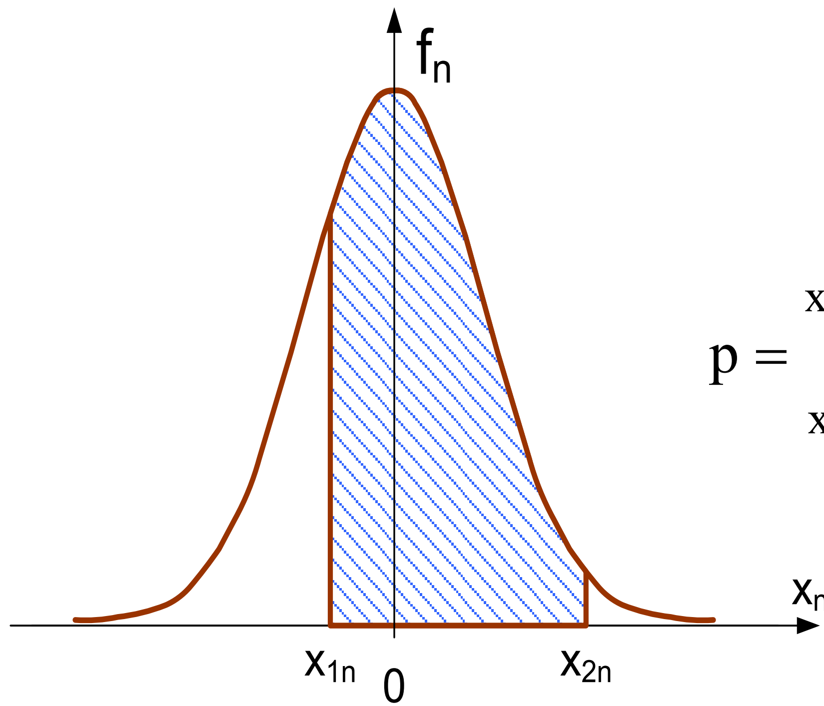


## *Background Information*

*Observation: The probability that the N(0,1) random variable $x_n$ satisfies the relationship $x_{1n}<x_n<x_{2n}$ is also given by*

$$p = F_n\left(x_{2n}\right) - F_n\left(x_{1n}\right)$$

*where $F_n(x)$ is the CDF of $x_n$.*




$$p = \int_{x_{1n}}^{x_{2n}} f_n\left(x\right)dx$$

*Since the N(0,1) distribution is symmetric around 0, p can also be expressed as*

$$p = F_n\left(x_{2n}\right) - \left(1 - F_n\left(-x_{1n}\right)\right)$$

*Background Information*

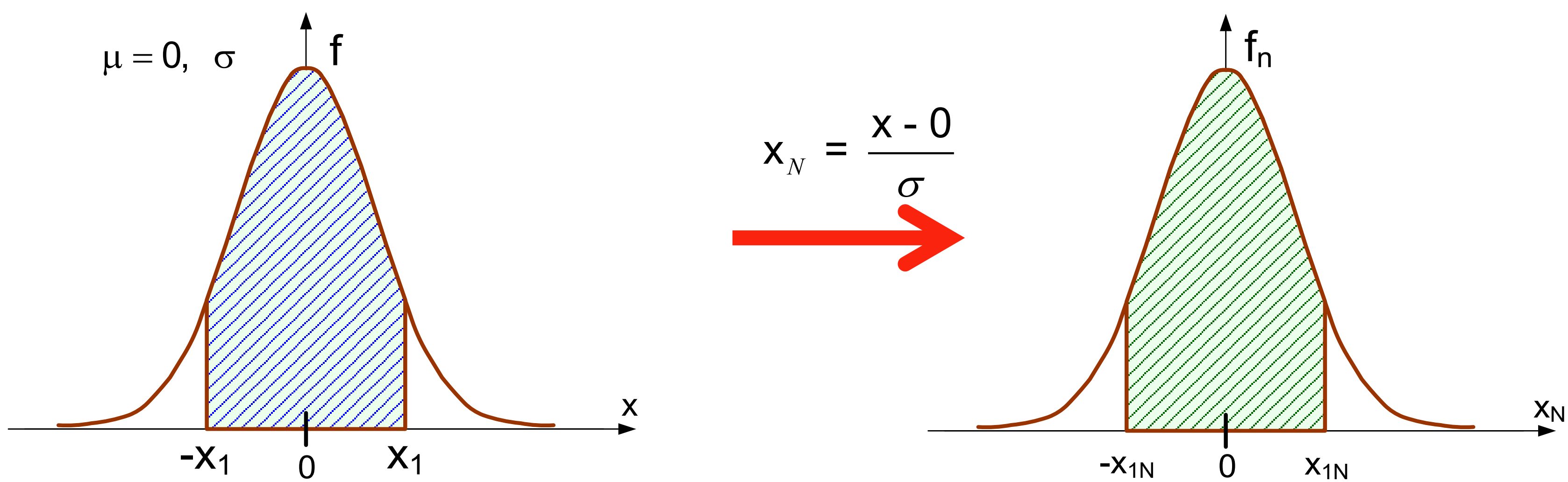
*Observation: In many electronic circuits, a random variable of interest, x, is 0 mean Gaussian and the probabilities of interest are characterized by a region defined by the* *<u>magnitude</u>* *of the random variable (i.e.* $-x_1 < x < x_1$*).*

*In these cases, if we define* $x_N = \dfrac{x - 0}{\sigma}$ then $x_N$ is N(0,1) and

$$p\left(-x_1 < x < x_1\right) = \int_{-x_1}^{x_1} f\left(x\right)dx = \int_{-x_{1n}}^{x_{1n}} f_n\left(x\right)dx = F_n\left(x_{1n}\right) - F_n\left(-x_{1n}\right)$$

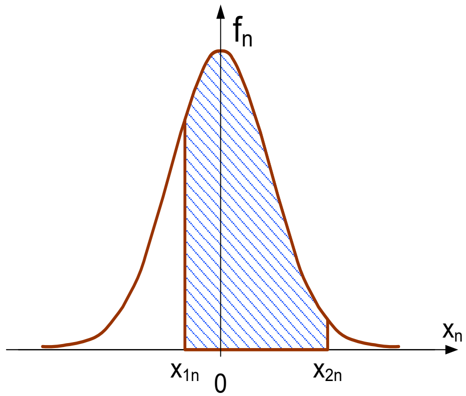
But for the N(0,1) distribution $F_n\left(-x_{1n}\right) = 1 - F_n\left(x_{1n}\right)$

therefore: $p = 2F_n\left(x_{1n}\right) - 1$

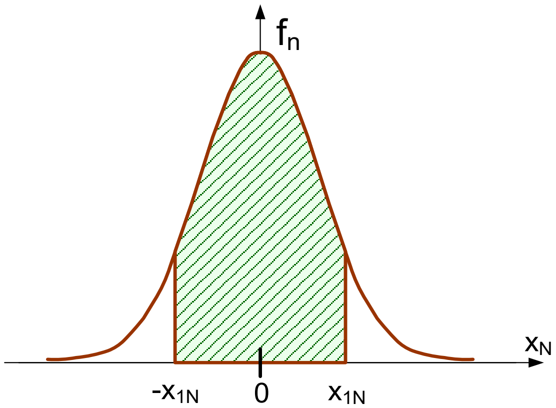


*Background Information*

$$p = F_n\left(x_{2n}\right) - F_n\left(x_{1n}\right)$$



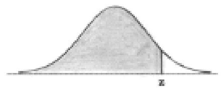

$$p = 2F_n\left(x_{1n}\right) - 1$$




Regardless of whether Gaussian performance requirements are asymmetric or symmetric, the CDF of the N(0,1) distribution (i.e. $F_n(x_n)$) is used to characterize yield

*Background Information*

*Tables of the CDF of the N(0,1) random variable are readily available.  It is also available in Matlab, Excel, and a host of other sources.*




**Probability Content from -oo to Z**

| Z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |

http://www.math.unb.ca/~knight/utility/NormTble.htm

*Background Information*

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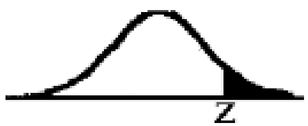


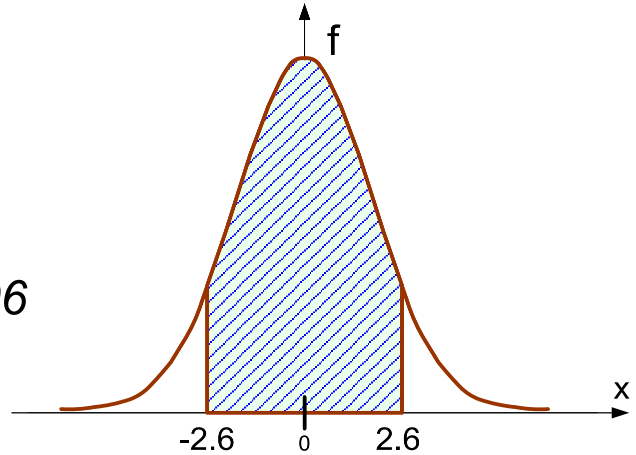

**Far Right**
**Tail Probabilities**

| Z | P{Z to oo} | Z | P{Z to oo} | Z | P{Z to oo} | Z | P{Z to oo} |
|---|---|---|---|---|---|---|---|
| 2.0 | 0.02275 | 3.0 | 0.001350 | 4.0 | 0.00003167 | 5.0 | 2.867 E-7 |
| 2.1 | 0.01786 | 3.1 | 0.0009676 | 4.1 | 0.00002066 | 5.5 | 1.899 E-8 |
| 2.2 | 0.01390 | 3.2 | 0.0006871 | 4.2 | 0.00001335 | 6.0 | 9.866 E-10 |
| 2.3 | 0.01072 | 3.3 | 0.0004834 | 4.3 | 0.00000854 | 6.5 | 4.016 E-11 |
| 2.4 | 0.00820 | 3.4 | 0.0003369 | 4.4 | 0.000005413 | 7.0 | 1.280 E-12 |
| 2.5 | 0.00621 | 3.5 | 0.0002326 | 4.5 | 0.000003398 | 7.5 | 3.191 E-14 |
| 2.6 | 0.004661 | 3.6 | 0.0001591 | 4.6 | 0.000002112 | 8.0 | 6.221 E-16 |
| 2.7 | 0.003467 | 3.7 | 0.0001078 | 4.7 | 0.000001300 | 8.5 | 9.480 E-18 |
| 2.8 | 0.002555 | 3.8 | 0.00007235 | 4.8 | 7.933 E-7 | 9.0 | 1.129 E-19 |
| 2.9 | 0.001866 | 3.9 | 0.00004810 | 4.9 | 4.792 E-7 | 9.5 | 1.049 E-21 |

## *Background Information*

*Example: Determine the probability that the N(0,1) random variable has magnitude less than 2.6*

$$p = 2F_n(2.6) - 1$$




*From the table of the CDF, $F_n(2.6) = 0.9953$ so $p=.9906$*

**Probability Content from -oo to Z**

| Z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |

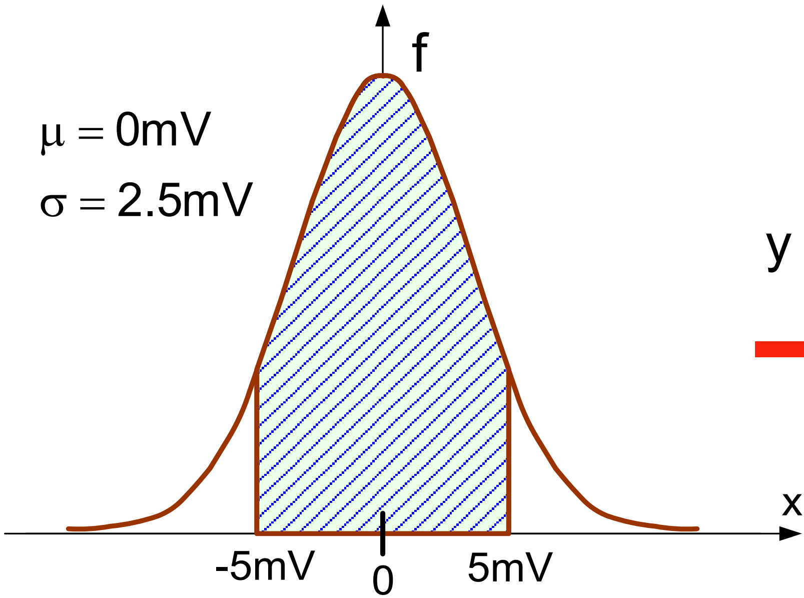
*Background Information*

It can be shown that the circuit designer has control of the offset voltage of an op amp and through architecture and sizing of devices can set the standard deviation of the offset voltage at almost any level. Invariably low offset voltages require larger area.

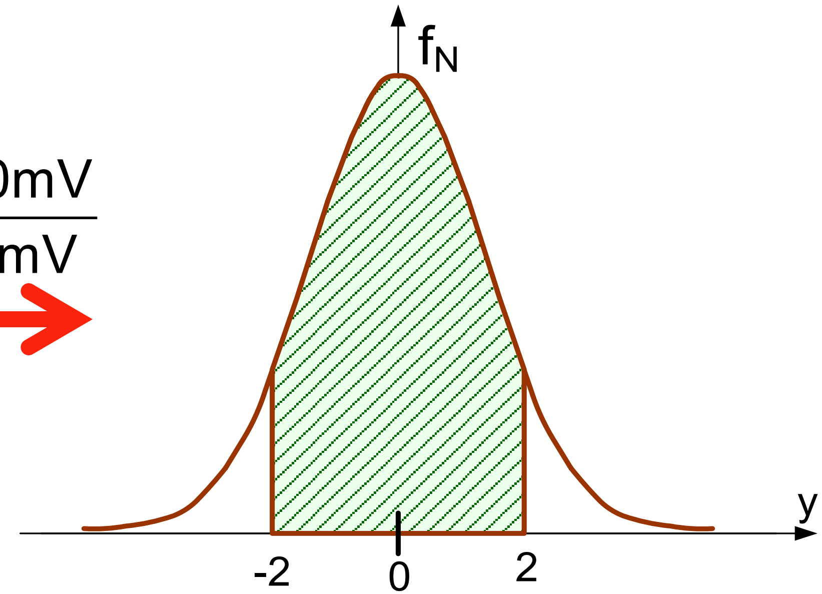
*Example: Determine the soft yield of an operational amplifier that has an offset voltage requirement of 5mV if the offset voltage has a Gaussian distribution with a standard deviation of 2.5mV and a mean of 0V.*

$\mu = 0\text{mV}$

$\sigma = 2.5\text{mV}$




$$y = \frac{x - 0\text{mV}}{2.5\text{mV}}$$

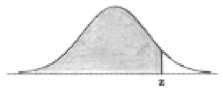



$$p = \int_{-2}^{2} f_N(x)dx = F_N(2) - F_N(-2) = 2 * F_N(2) - 1$$

$$p = 2 * F_N(2) - 1$$

*Background Information*

## Example (continued)



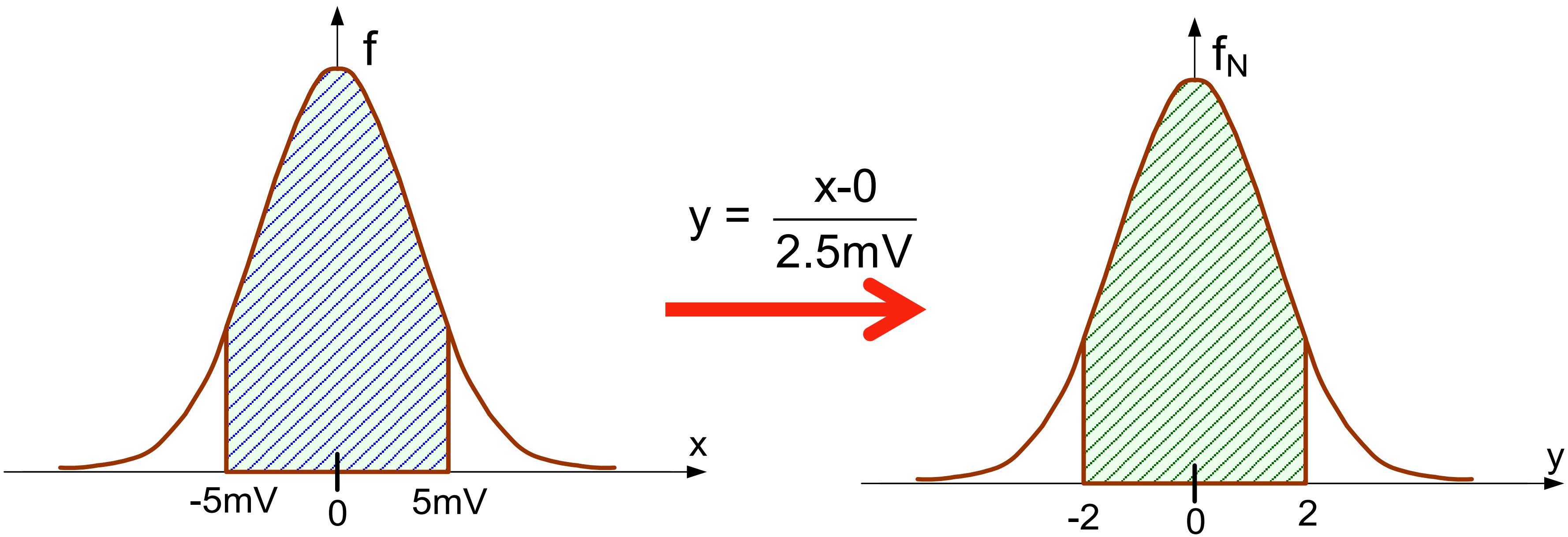

**Probability Content**
**from -oo to Z**

| Z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |

http://www.math.unb.ca/~knight/utility/NormTble.htm

Example (continued)

*Determine the soft yield of an operational amplifier that has an offset voltage requirement of 5mV if the offset voltage has a Gaussian distribution with a standard deviation of 2.5mV and a mean of 0V.*




$$p = 2 * F_N(2) - 1$$

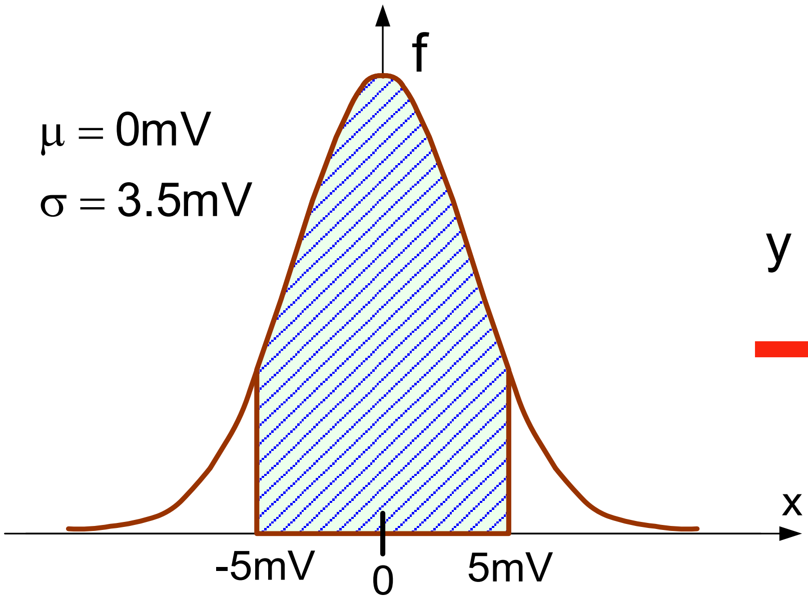
$$F_N(2) = 0.9772$$

$$p = 2 * .9772 - 1 = .9544$$

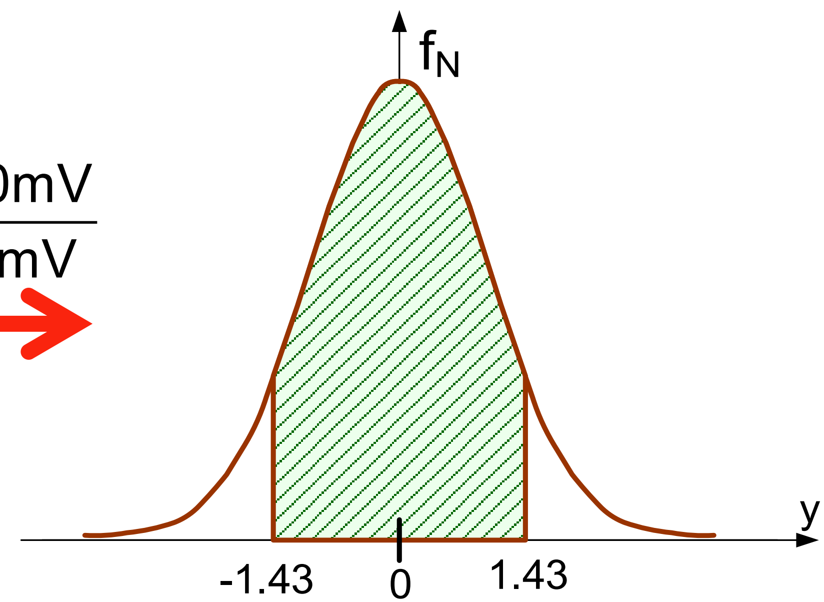
*Background Information*

Repeat the previous example if the designer decided to reduce the area so that the standard deviation increased to 3.5 mV

*Example: Determine the soft yield of an operational amplifier that has an offset voltage requirement of 5mV if the offset voltage has a Gaussian distribution with a standard deviation of 3.5mV and a mean of 0V.*




$$y = \frac{x - 0mV}{3.5mV}$$

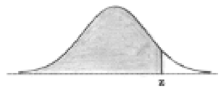



$$p = \int_{-1.43}^{1.43} f_N(x)dx = F_N(1.43) - F_N(-1.43) = 2 * F_N(1.43) - 1$$

$$p = 2 * F_N(1.43) - 1$$

# *Background Information*

## Example (continued)




**Probability Content from -oo to Z**

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| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
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| 1.6 | 0.9452 | 0.9463 | [illegible] | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | [illegible] | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
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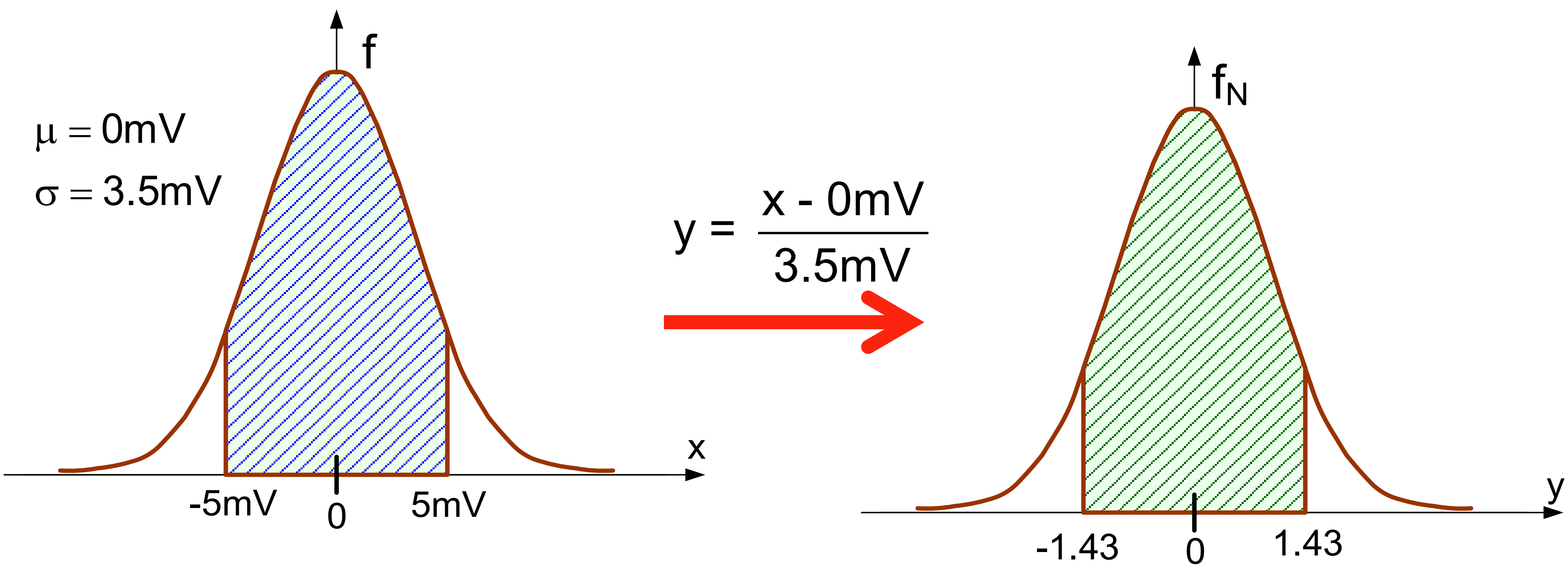
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |

http://www.math.unb.ca/~knight/utility/NormTble.htm

*Background Information*

Repeat the previous example if the designer decided to reduce the area so that the standard deviation increased to 3.5 mV

*Example: Determine the soft yield of an operational amplifier that has an offset voltage requirement of 5mV if the offset voltage has a Gaussian distribution with a standard deviation of 3.5mV and a mean of 0V.*

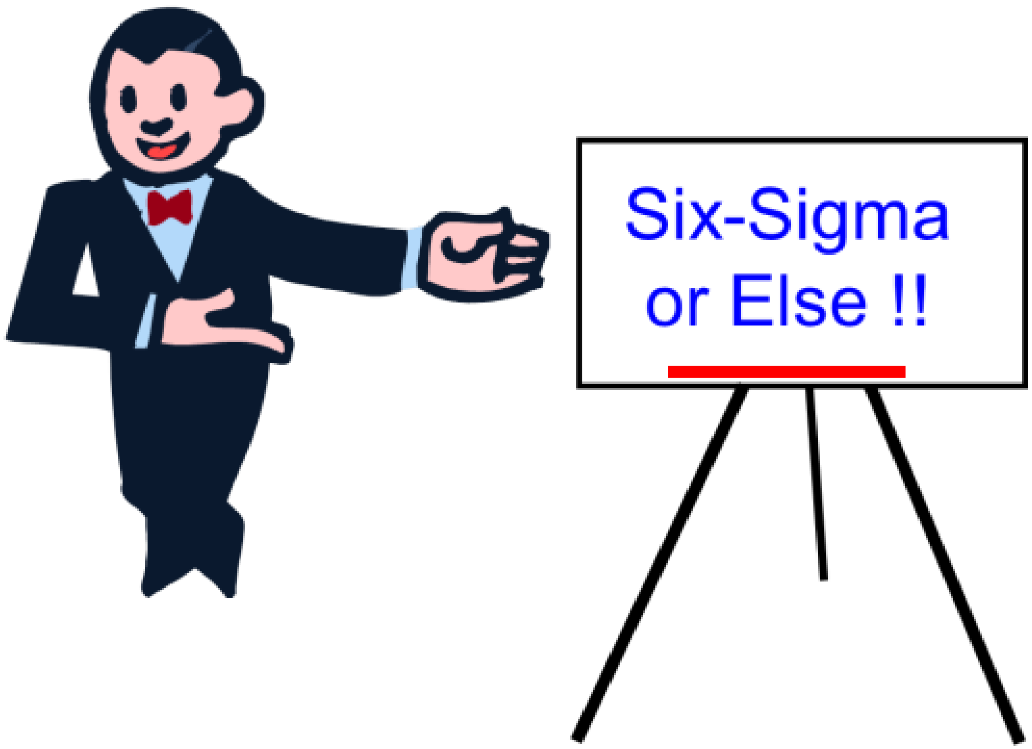



$$p = 2 * F_N(1.43) - 1 = 2 * 0.9236 - 1 = 0.847$$

This small change in the design dropped the yield from just over 95% to just under 85%

Statistical analysis is critical for predicting performance capabilities of many ICs !

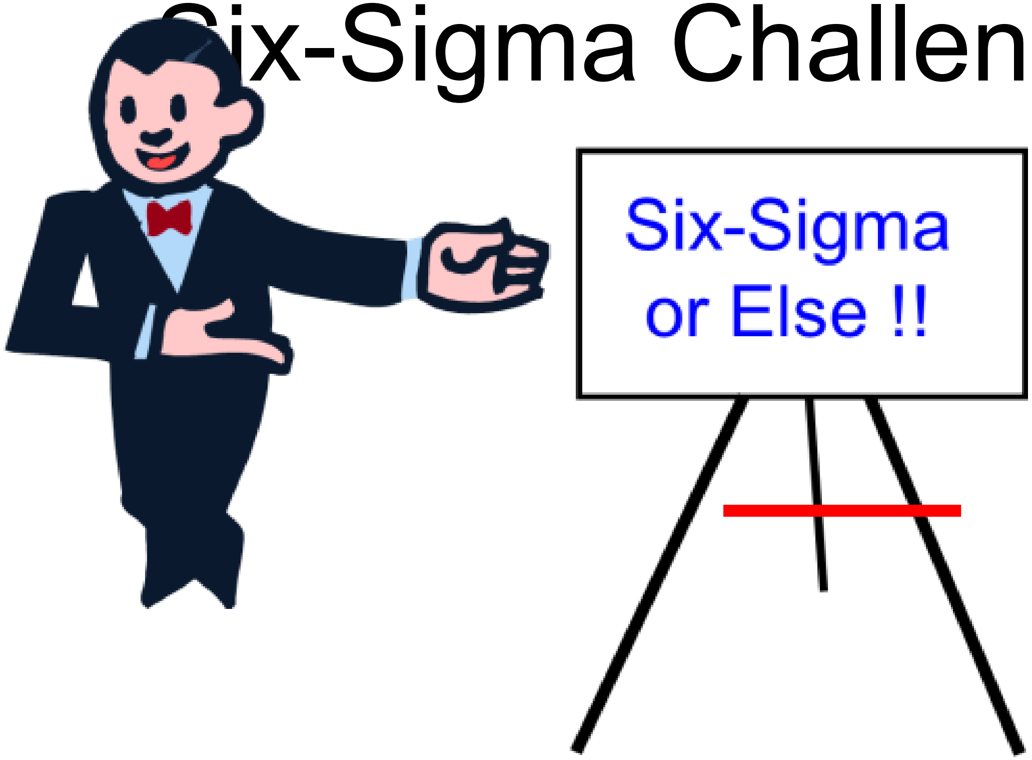
# Many Companies Promote the Real Six-Sigma Challenge




From Wikipedia Sept 1 2021

**Six Sigma** (**6σ**) is a set of techniques and tools for process improvement. It was introduced by American engineer Bill Smith while working at Motorola in 1986.[1][2] A six sigma process is one in which 99.99966% of all opportunities to produce some feature of a part are statistically expected to be free of defects.

# Many Companies Promote the Real Six-Sigma Challenge



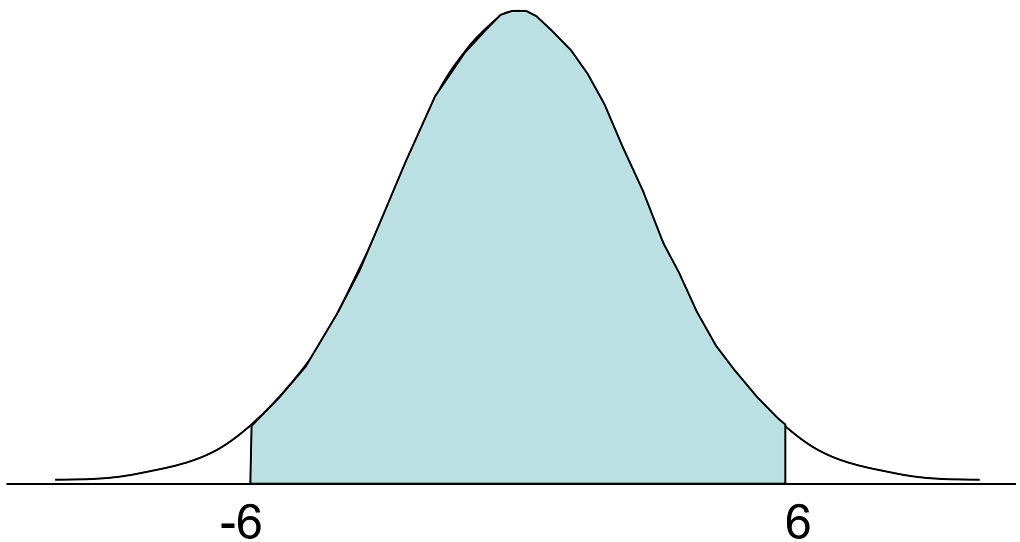

From Wikipedia   Sept 1  2021

In 2005 Motorola attributed over $17 billion in savings to Six Sigma.[3]

By the late 1990s, about two-thirds of the Fortune 500 organizations had begun Six Sigma initiatives with the aim of reducing costs and improving quality.[6]

# Yield at the Six-Sigma level

(Assume a Gaussian distribution)

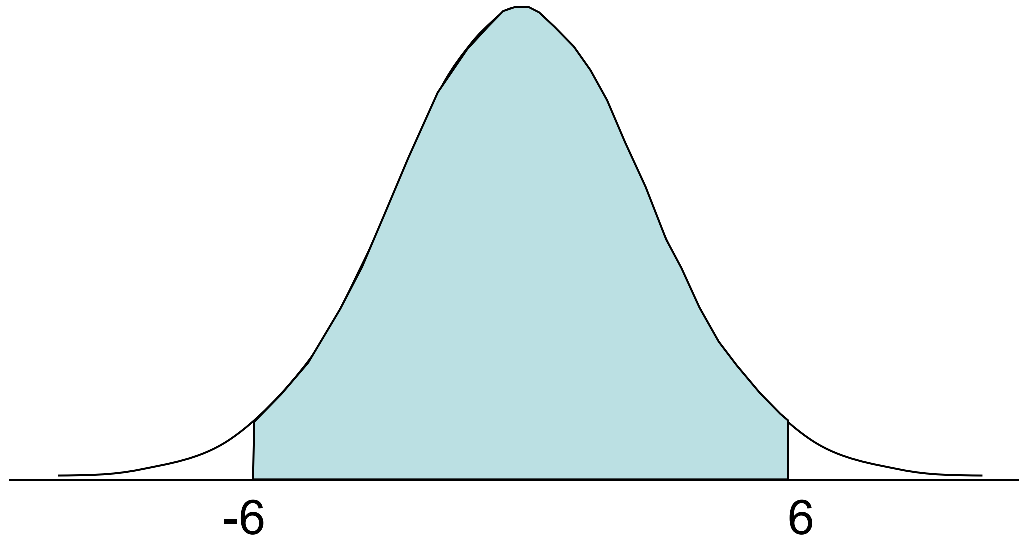



$$Y_{6sigma} = 2F_N(6) - 1$$

$$Y_{6sigma} = 0.9999999980$$

This is approximately 2 defects out of 1 billion parts

# Yield at the Six-Sigma level




This is approximately 2 defects out of 1 billion parts

Would producing ICs with a yield at the six-sigma level be a good goal?

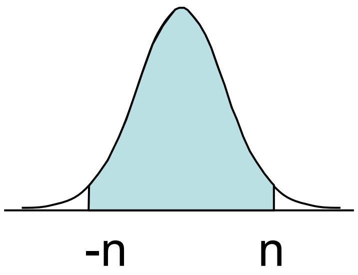
How about smart phones with defects at this level? (approx. 1.4B sold in 2020)

How about automobiles? (approx. 78 million produced in 2020)

# Six-Sigma or Else !!

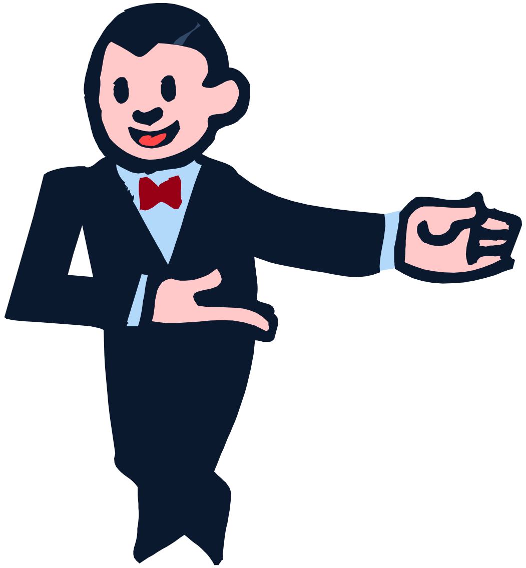
How serious is the “or Else” in the six-sigma programs?

# Yield at Various Sigma Levels




| No Sigma | Yield | Defect Rate |
|---|---|---|
| 1 | 0.682689492 | 0.317311 |
| 2 | 0.954499736 | 0.0455 |
| 3 | 0.997300204 | 0.0027 |
| 4 | 0.999936658 | 6.33E-05 |
| 5 | 0.999999427 | 5.73E-07 |
| 6 | 0.9999999980 | 1.97E-09 |
| 7 | 0.9999999999974 | 2.56E-12 |

Six-sigma performance is approximately 2 defects in a billion !



# Six-Sigma or Else !!

It is assumed that the performance or yield will drop, <u>for some reason</u>, by 1.5 sigma after a process has been established

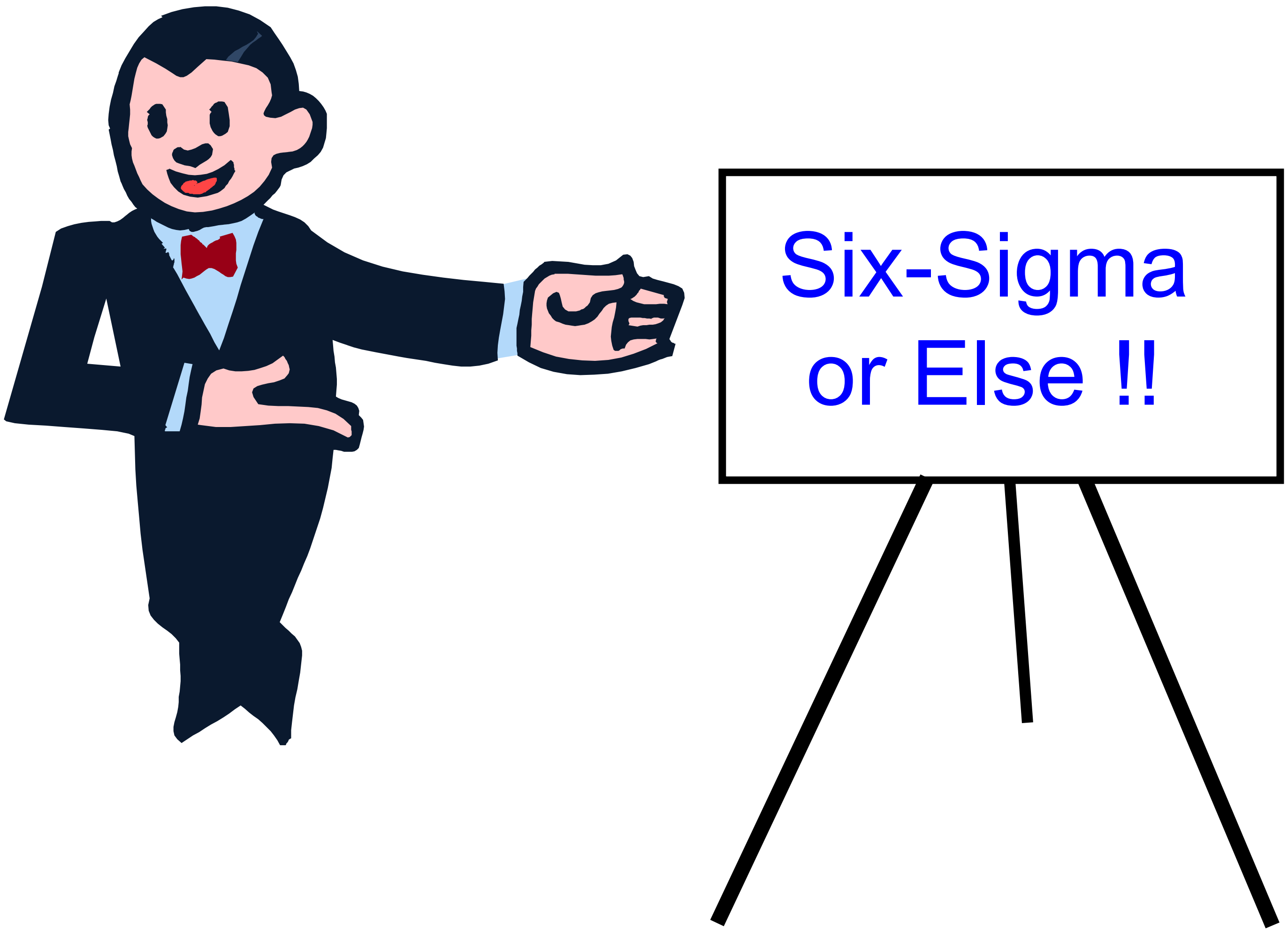
Initial "six-sigma" solutions really expect only 4.5 sigma performance in steady-state production

Assumption : Processes of interest are Gaussian (Normal)

4.5 sigma performance corresponds to 3.4 defects in a million

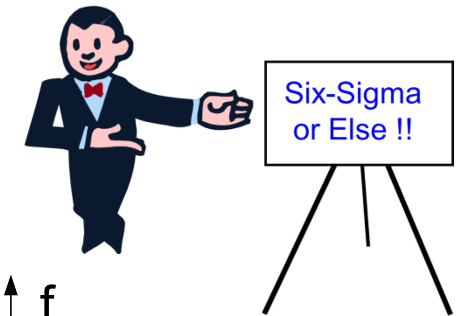
Observation: Any Normally distributed random variable can be mapped to a N(0,1) random variable by subtracting the mean and dividing by the variance

# Meeting the Real Six-Sigma Challenge

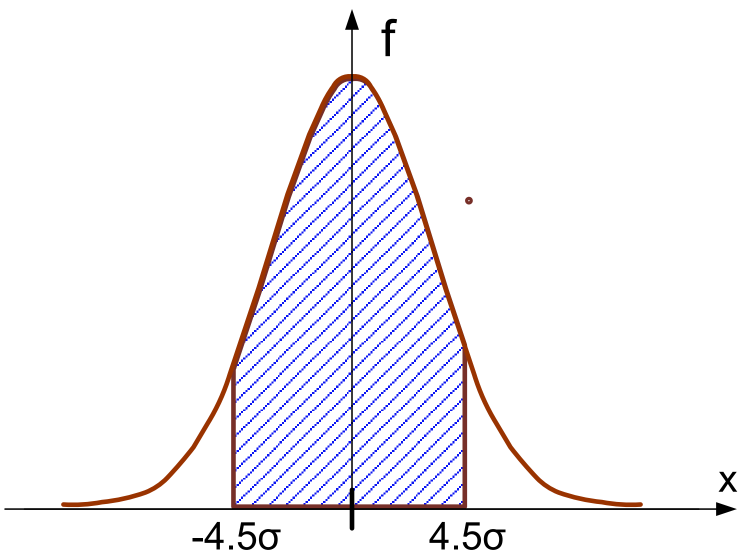


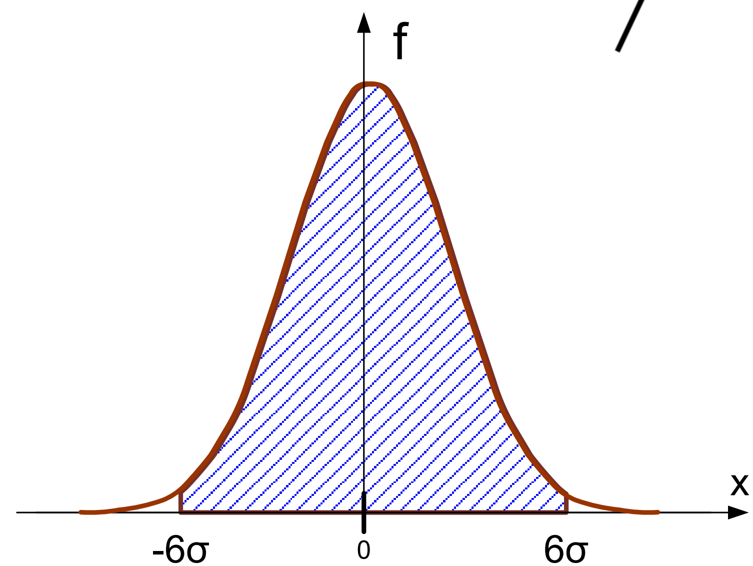

Highly Statistical Concept !

# The Six-Sigma Challenge




## Two-sided capability:




Long-term Capability

Tails are 6.8 parts in a million

Short-term Capability

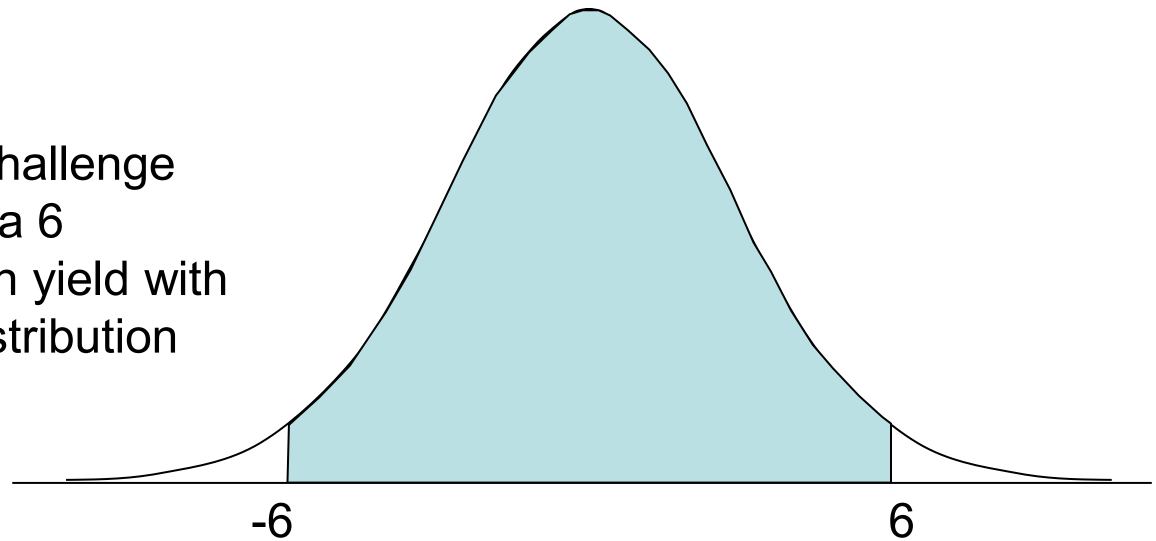
Tail is 2 parts in a billion

**Six Sigma Performance is Very Good !!!**

Example: Determine the maximum die area if the circuit yield is to initially meet the "six sigma" challenge for hard yield defects (Assume a defect density of $1cm^{-2}$ and only hard yield loss). Is it realistic to set six-sigma die yield expectations on the design and process engineers?

Solution:

The "six-sigma" challenge requires meeting a 6 standard deviation yield with a Normal (0,1) distribution




$$Y_{6sigma} = 2F_N(6) - 1$$

Recall: $F_N(6)=0.9999999980$

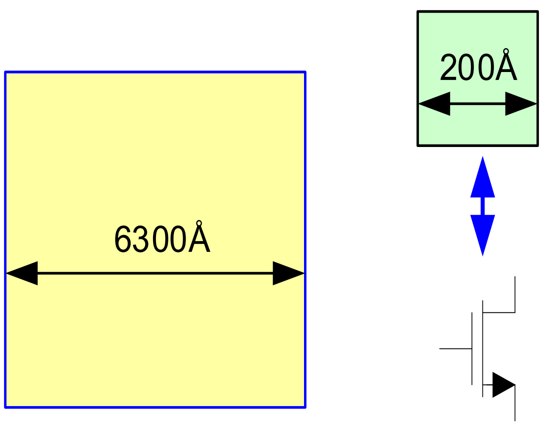
$$Y_{6sigma} = 0.999999996$$

Solution cont:

$$Y_H = e^{-Ad}$$

$$A = \frac{-\ln(Y_H)}{d}$$

$$A = \frac{-\ln(.9999999980)}{1\text{cm}^{-2}} = 4.0E-9\text{cm}^2 = 40E6(\mathring{A})^2$$




This is comparable to the area required to fabricate about 100 transistors in a state of the art 20nm process (assuming 10x overhead)

Solution cont:

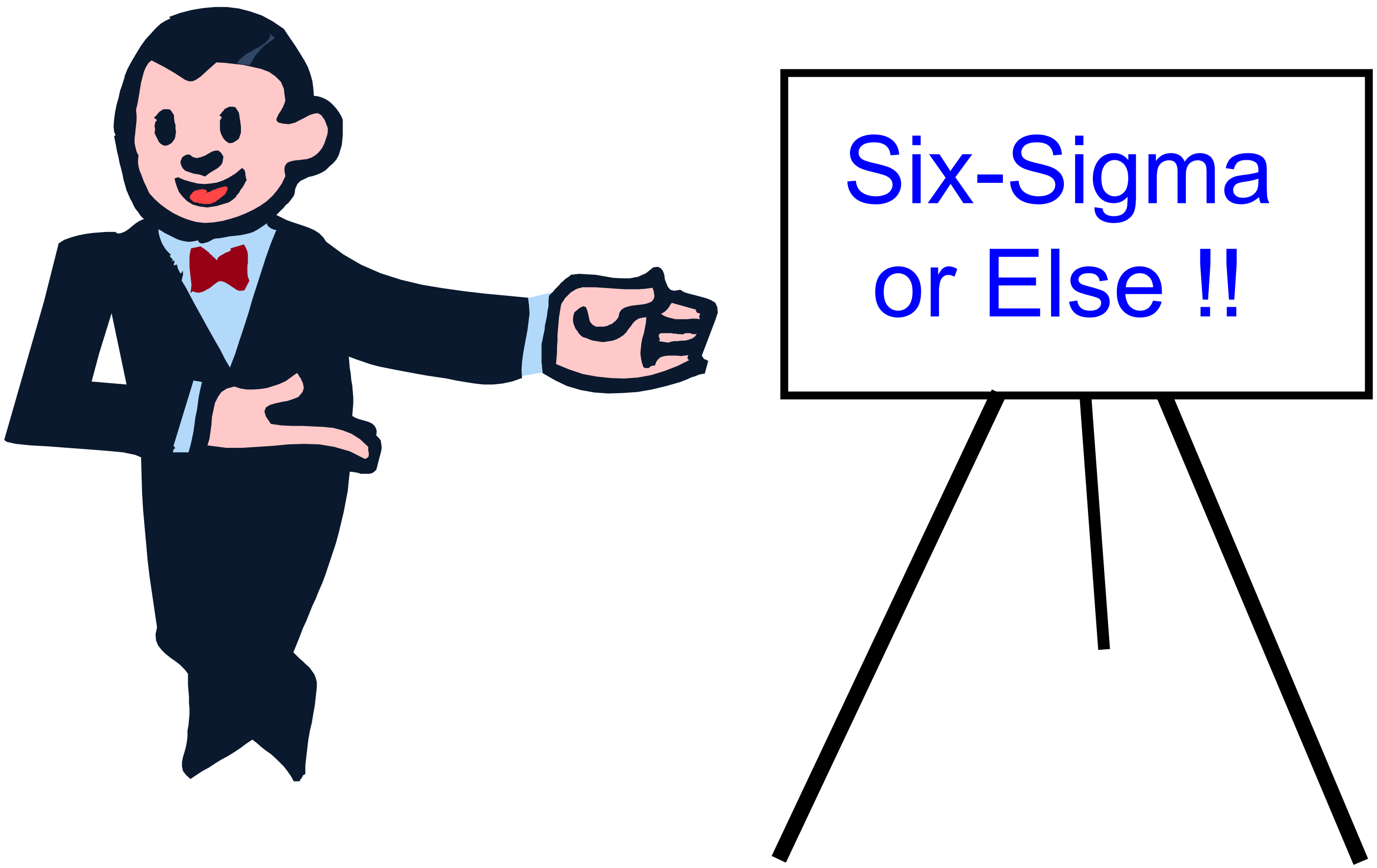
Is it realistic to set six-sigma die hard yield expectations on the design and process engineers?

The best technologies in the world have orders of magnitude too many defects to build any useful integrated circuits with die yields that meet six-sigma performance requirements !!

**Arbitrarily setting six-sigma design requirements will guarantee financial disaster !!**

# Meeting the Real Six-Sigma Challenge

# Meeting the Real Six-Sigma Challenge




**Improving a yield by even one sigma often is VERY challenging !!**

# Statistics can be abused !

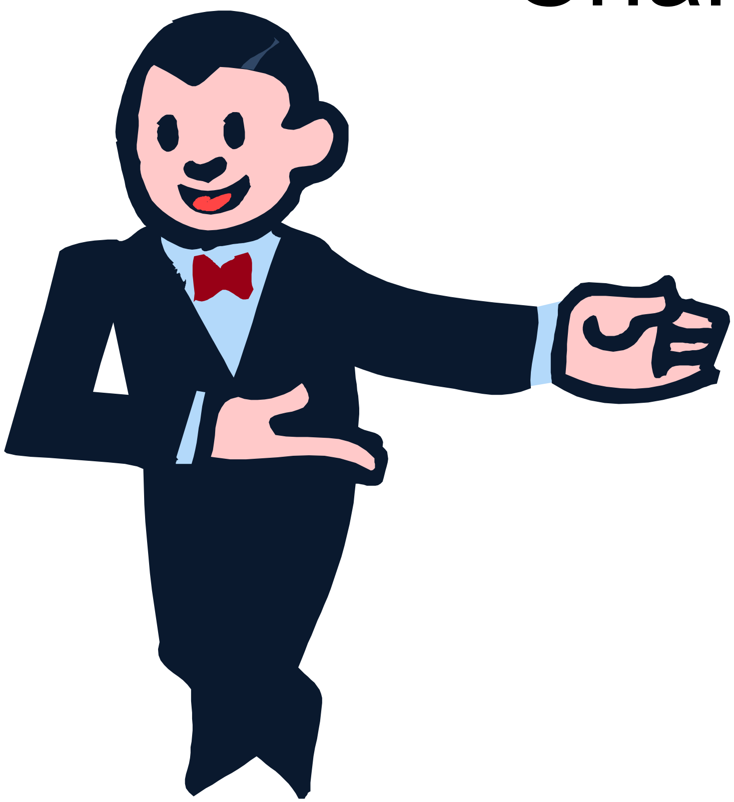
Many that are not knowledgeable incorrectly use statistics

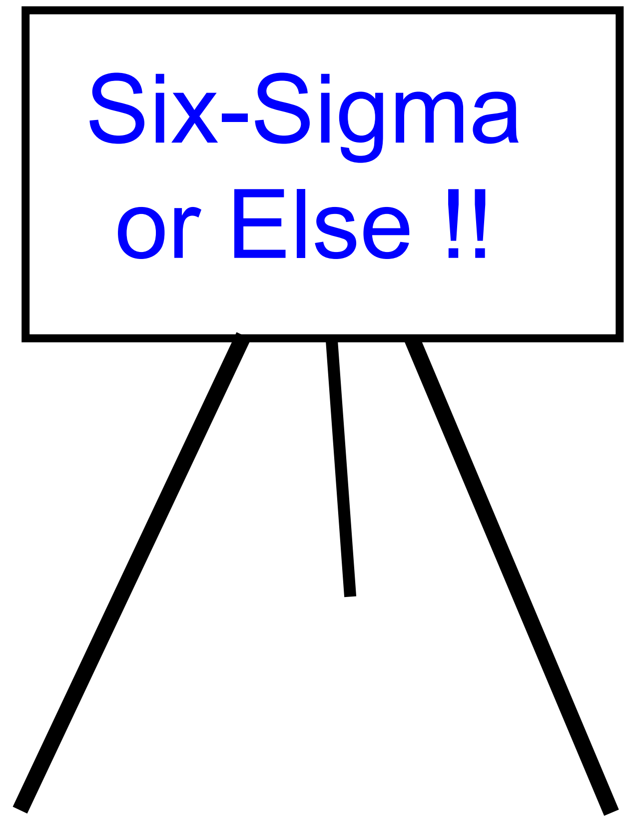
Many use statistics to intentionally mislead the public

Some openly abuse statistics for financial gain or for manipulation purposes

Keep an open mind to separate "good" statistics from "abused" statistics

# Meeting the Real Six-Sigma Challenge

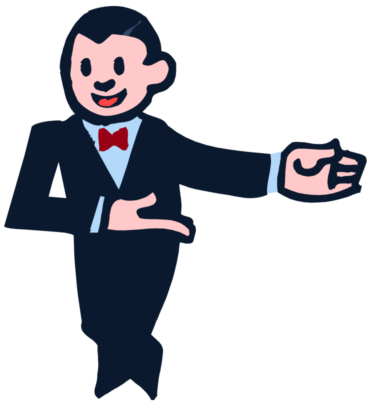





How has Motorola fared with the 6-sigma approach?

**Motorola, Inc.** (pronounced ) was an American multinational[6] telecommunications company based in Schaumburg, Illinois, which was eventually divided into two independent public companies, Motorola Mobility and Motorola Solutions on January 4, 2011, after losing $4.3 billion from 2007 to 2009.[7]

# Meeting the Real Six-Sigma Challenge



How has Motorola fared with the 6-sigma approach?

| | MOTOROLA |
|---|---|
| Former type | Public company |
| Industry | Telecommunications |
| Fate | Divided into Motorola Mobility and Motorola Solutions |
| Successor | Motorola Mobility<br>Motorola Solutions |
| Founded | September 25, 1928 |
| Defunct | January 4, 2011 |

- Sold military activities to General Dynamics 2000/2001
- Sold automotive products in 2006
- Spun of discrete components as ON semiconductor in 1999
- Spun of SPS as Freescale in 2003 - Acquired by NXP in 2015
- Sold Motorola Mobility to Google in 2011 – Acquired by Lonovo in 2014
- Motorola Solutions has 16,000 employees (ref fall 2018) , down from over 150,000 in mid ‘90s

# The "Motorola" saga continues

Freescale Semiconductor
Semiconductor manufacturing company

Freescale Semiconductor, Inc. was an American multinational corporation headquartered in Austin, Texas, with design, research and development, manufacturing and sales operations in more than 75 locations in 19 countries. Wikipedia

**Headquarters:** Austin, TX

**CEO:** Gregg A. Lowe (Jun 2012–)

**Number of employees:** 17,300 (2013)

**Defunct:** December 7, 2015

**Parent organization:** Freescale Semiconductor Holdings I Ltd

**Subsidiaries:** Freescale Semiconductor Japan Ltd, MORE

Qualcomm, NXP strike $38B semiconductor deal | PitchBook
https://pitchbook.com/news/articles/qualcomm-nxp-strike-38b-semiconductor-deal
Oct 27, 2016 - **Qualcomm** has agreed to acquire **NXP** Semiconductors for $110 per ... The **deal** represents an enterprise value of $47 billion and an equity ...

Trump Blocks Broadcom's Bid for Qualcomm - The New York Times
https://www.nytimes.com/2018/03/12/.../trump-broadcom-qualcomm-merger.html
Mar 12, 2018 - Image. **Broadcom** had been trying for months to buy **Qualcomm**, and .... change the world of **mergers** and acquisitions and open the door to the ...

Will China Approve Qualcomm's NXP Acquisition? - Forbes
https://www.forbes.com/sites/.../05/.../will-china-approve-qualcomms-nxp-acquisition/
May 16, 2018 - Qualcomm's deal to purchase **NXP** Semiconductors has been caught in the crosshairs of the trade tensions between the U.S. and China, with ...

Chinese regulators approve Qualcomm purchase of NXP for US$44 ...
https://www.scmp.com › Business › Companies
Jun 15, 2018 - Chinese regulators have approved US semiconductor company Qualcomm's proposed US$44 billion **acquisition** of Dutch chip maker **NXP** ...

Qualcomm drops NXP acquisition, leaves analysts concerned about ...
https://www.marketwatch.com › Industries › The Ratings Game
Jul 26, 2018 - Nearly two years after Qualcomm Inc. announced its intent to **acquire NXP** Semiconductors NV, investors are pleased that the company is ...

Stay Safe and Stay Healthy !

# End of Lecture 4